3-ph induction motor

This motor is also called as asynchronous motor because it runs at a speed less than synchronous speed.it is a singly excited machine.

3-ph I.M is most popular type of a.c motor. It is most commonly used for industrial drives since it is cheap, robust, efficient and reliable. It has good speed regulation and high starting torque. it requires less maintenance.

Construction

Induction motor have two main parts namely rotor and stator

1. Stator: it is a stationary part of induction motor.it is built up of high-grade alloy steel laminantions to reduce eddy-current losses.the laminations are slotted on the inner periphery and are insulated from each other. these laminations are supported in a stator frame of cast iron. The insulated stator conductors are placed in these slots. the insulated stator conductors are connected to form a 3-ph winding, and the three phase supply is given to it.

2. Rotor: The rotor is a rotating part of induction motor. It is also built of thin laminations of the same material as the stator. The rotor is connected to the mechanical load through the shaft.

The rotor of the three phase induction motor are further classified as

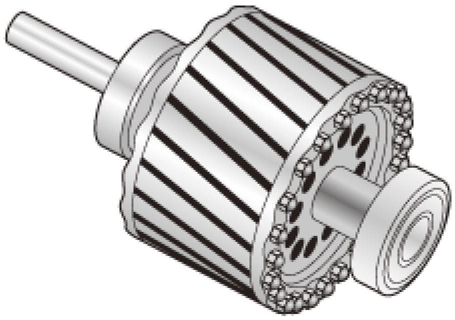
1. Squirrel cage rotor or cage rotor,
2. Slip ring rotor or wound rotor or phase wound rotor.

Depending upon the type of rotor construction used the three phase induction motor are classified as:

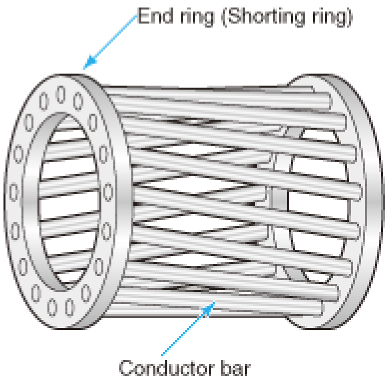
1. Squirrel cage induction motor,
2. Slip ring induction motor or wound induction motor or phase wound induction motor

Squirrel cage Rotor:

The rotor core is a cylindrical laminated iron core , with slots around core carrying rotor conductors. Steel laminations are provided on the rotor core. Each slot contains an uninsulated bar conductors of alumunium or copper. At the end of each rotor, the rotor bar conductors are short circuited by heavy end rings. These two components – conductors and end rings form the structure of a squirrel cage , that’s why the rotor is named so.In motors upto 100kW rating the squirrel cage structure is made up of cast alumunium. The rotor conductor bars are not exactly parallel to the shaft but they are slightly skewed.



(a) Rotor



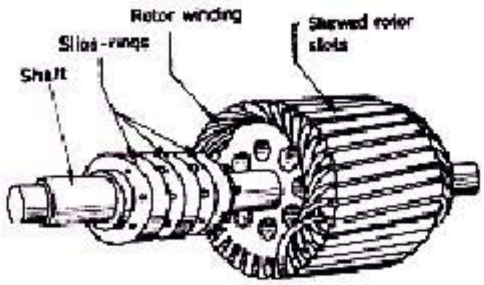
(b) Squirrel-cage conductor

Skewing of squirrel cage rotor offers some advantages. They are:

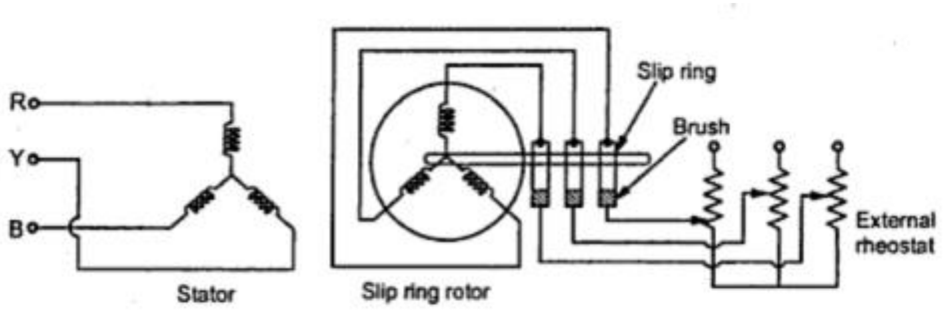
1. More uniform torque is produced.
2. Noise (humm) is reduced.
3. Locking tendency of rotor i.e cogging is reduced.

Slip ring rotor or wound rotor

The wound rotor consists of a slotted armature. Insulated conductors are put in the slots and connected to form a three – phase double layer distributed winding similar to stator winding. The number of poles of rotor are kept same to the number of poles of the stator.the rotor windings are connected in star.the open ends of the star circuit are brought outside the rotor and connected to three insulated slip rings. The slip rings are mounted on the shaft with brushes resting on them. The brushes are connected to three variable resistors connected in star. This arrangement is done to introduce an external resistance in rotor circuit for Starting purposes and for changing the speed-torque-characteristics. When motor is running at its rated speed, slip rings are automatically short circuited by means of a metal collar and brushes are lifted above the slip rings to minimize the frictional losses.



Slip ring rotor



Comparison between cage and wound rotors

Squirrel cage induction motor	**Slip ring or phase wound Induction motor**
Construction is very simple	Construction is complicated due to presence of slip ring and brushes
The rotor consists of rotor bars which are permanently shorted with the help of end rings	The rotor winding is similar to the stator winding
Since the rotor bars are permanently shorted, its not possible to add external resistance	We can easily add rotor resistance by using slip ring and brushes
Staring torque is low and cannot be improved	Due to presence of external resistance high starting torque can be obtained
Slip ring and brushes are absent	Slip ring and brushes are present
Less maintenance is required	Frequent maintenance is required due to presence of brushes
Speed control by rotor resistance method is not possible	Speed control by rotor resistance method is possible
Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc	Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc

Operating principle of squirrel cage induction motor

When the stator winding of a 3 phase induction motor is connected to a 3 phase a.c supply, a rotating magnetic field (rmf) is established in stator core which rotates at synchronous speed. The direction of revolution of this field will depend upon the phase sequence of the current i.e the order of connection of stator terminals to the supply.

Due to presence of rmf the flux is cut by the rotor conductors and by the principle of electromagnetic induction the emf is induced in the conductors of rotor. As the rotor ends are short circuited by the end rings the circuit is complete this causes a current to flow in rotor conductors. We know that when a conductor carrying current is put in a magnetic field , a force is produced in it. Thus rotor conductors experiences the force whose direction is governed by Left hand rule. Torque is produced on all rotor conductors which rotates the rotor in same direction as that of rotating magnetic field. Hence three phase induction motor is self starting. and the motor is named so since its operation depends upon induced voltage in the rotor conductors.

Speed and slip of squirrel cage induction motor

As motor starts rotating its speed is less than synchronous speed and hence there is relative motion between rotor conductors & rotating field.
due to this emf is being induced and current is flowing in the conductors. due to flow of current conductors are continuously experiencing force in a direction of rmf. When speed of rotor reaches to Ns the relative motion becomes zero. Hence no induced emf, no current and no force on rotor conductors. so speed starts decreasing but as speed goes low again relative motion between rotor and rmf is established which again speed up the rotor. In this way rotor always tries to approach upto Ns, but it slightly lags by synchronous speed.
This difference is called **slip speed** and the phenomenon is called **slip.**

Slip speed (s) = $N_s - N_r$ in rpm
Fraction slip = $(N_s - N_r) / N_s$ in rpm
Percentage slip = Fraction slip × 100 in rpm

At no load the slip is very small and the motor runs at synchronous speed . As load is increased , load torque increases which increase the slip. At about full load slip varies form 2 – 5 %.

Effect of Slip on Rotor Parameters

1.Frequency of rotor:

We know that $N_s = 120f / P$
Frequency of current and voltage in stator is same as supply frequency ,
$f = PNs / 120$ ....(i)

Frequency of rotor depends upon slip and is given by
$f_r = P (N_s - N_r) / 120$ ....(ii)

Thus we get the relation between stator and rotor frequency and it is

$f_r = s\ f$. (Rotor current frequency = s × supply frequency)

when rotor is driven by a mechanical prime mover at synchronous speed Ns, then s = 0 & fr = 0.
Hence rotor frequency varies from fr = f to 0.

➔ $0 \le fr \le f$

$Nr = (1-s)Ns$ (Nr –motor speed)

2.Effect of Slip on Magnitude of Rotor Induced E.M.F

When rotor is standstill, s = 1, relative speed is maximum and maximum e.m.f. gets induced in the rotor. Let this e.m.f. be,

E_2 = Rotor induced e.m.f. per phase on standstill condition

As rotor gains speed, the relative speed between rotor and rotating magnetic field decreases and hence induced e.m.f. in rotor also decreases as it is proportional to the relative speed N_s - N. Let this e.m.f. be,

E_{2r} = Rotor induced e.m.f. per phase in running condition

Now $E_{2r} \alpha N_s$ while $E_{2r} \alpha N_s - N$

Dividing the two proportionality equations,

$E_{2r}/E_2 = (N_s - N)/N_s$ but $(N_s - N)/N$ = slip s

$E_{2r}/E_2 = s$

$E_{2r} = s\ E_2$

The magnitude of the induced e.m.f in the rotor also reduces by slip times the magnitude of induced e.m.f. at standstill condition.

3.Effect on Rotor Resistance and Reactance

The rotor winding has its own resistance and the inductance. In case of squirrel cage rotor, the rotor resistance is very very small and generally neglected but slip ring rotor has its own resistance which can be controlled by adding external resistance through slip rings. In general let,

R_2 = Rotor resistance per phase onstandstill

X_2 = Rotor reactance per phase on standstill

Now at standstill, fr = f hence if L_2 is the inductance of rotor per phase,

$X_2 = 2\pi fr\ L_2 = 2\pi f\ L_2$ Ω/ph

While R_2 = Rotor resistance in Ω/ph

Now in running condition, $f_r = s\ f$ hence,

$X_{2r} = 2\pi f_r L_2 = 2\pi fs\ L_2 = s\ .(2\pi f\ L_2)$

$X_{2r} = s\ X_2$

where X_{2r} = Rotor reactance in running condition

Thus resistance as independent of frequency remains same at standstill and in running condition. While the rotor reactance decreases by slip times the rotor reactance at standstill.

Hence we can write rotor impedance per phase as :

Z_2 = Rotor impedance on standstill (N = 0) condition

$= R_2 + j\, X_2\ \Omega$/ph

$Z_2 = \surd(R_2^2+ X_2)^2)\ \Omega$/ph ...... magnitude

While Z_{2r} = Rotor impedance in running condition

$= R_2 + j\, X_{2r} = R_2 + j\, (s\, X_2)\ \Omega$/ph

$Z_{2r} = \surd(R_2^2+ (s\, X_2)^2)\ \ \Omega$/ph ...... magnitude

4. Effect on Rotor Power Fcator

From rotor impedance, we can write the expression for the power factor of rotor at standstill and also in running condition.

The impedance triangle on standstill condition is shown in the Fig1. From it we can write,

$\cos \Phi_2$ = Rotor power factor on standstill

$= R_2/Z_2 = R_2/\surd(R_2^2+ X_2^2)$

The impedance in running condition becomes Z_{2r} and the corresponding impedance triangle is shown in the Fig.2. From Fig. 2 we can write,

$\cos \Phi_2 r$ = Rotor power factor in running condition

$= R_2/Z_{2r} = R_2/\surd(R_2^2+ (s\, X_2)^2)$

Key point : As rotor winding is inductive, the rotor p.f. is always lagging in nature.

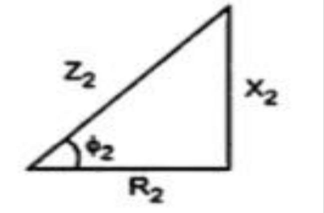


Fig. 1

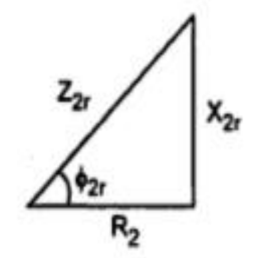


Fig. 2

5. Effect on Rotor Current

Let I_2 = Rotor current per phase on standstill condition

The magnitude of I_2 depends on magnitude of E_2 and impedance Z_2 per phase.

I_2 = (E_2 per phase)/(Z_2 per phase) A

Substituting expression of Z_2 we get,

$$I_2 = E_2 /\sqrt{(R_2^2 + X_2^2)} \quad A$$

The equivalent rotor circuit on standstill is shown in the Fig.3. The Φ_2 is the angle between E_2 and I_2 which determines rotor p.f. on standstill.

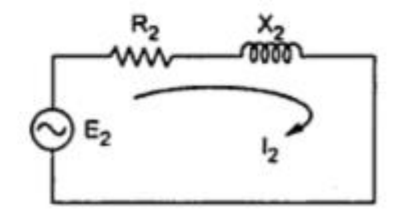


Fig. 3

In the running condition, Z_2 changes to Z_{2r} while the induced e.m.f. changes to E_{2r}. Hence the magnitude of current in the running condition is also different than on standstill. The equivalent circuit on running condition is shown in the Fig. 4.

I_{2r} = Rotor current per phase in running condition

The value of slip depends on speed which inturn depends on load on motor hence X_{2r} is shown variable in the equivalent circuit. From the equivalent we can write,

$$I_{2r} = E_{2r}/Z_{2r} = (s\,E_2)/\sqrt{(R_2^2 + (s\,X_2)^2)}$$

Φ_{2r} is the angle between E_{2r} and I_{2r} which decides p.f. in running condition .

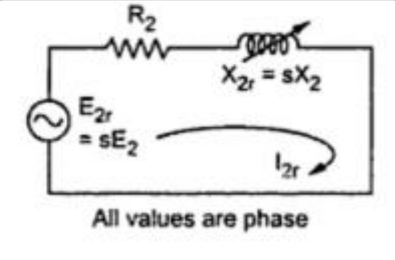


Fig. 4

Key point : Putting s = 1 in the expression obtained in running condition, the values at standstill can be obtained.

Relation between rotor copper loss and rotor input

Let T = developed torque = torque exerted on rotor by rotating flux

P_g = power transferred from stator to rotor = air gap power = input power to rotor

P_c = rotor copper loss

P_m = mechanical power developed by rotor

Now power input to the rotor P_g is from stator side through rotating magnetic field which is rotating at synchronous speed N_s

$P_g = T \times \omega_s$ where $\omega_s = (2\pi N_s)/60$ rad/sec

$P_g = T \times (2\pi N_s)/60$ where N_s is in r.p.m. …………(1)

Total mechanical power developed by rotor

$P_m = T \times \omega_r$ where $\omega_r = (2\pi N_r)/60$

$P_m = T \times (2\pi N_r)/60$ …………(2)

The difference between P_g and P_m is rotor copper loss P_c

$P_c = P_2 - P_m = T \times (2\pi N_s/60) - T \times (2\pi N/60)$

$P_c = T \times (2\pi/60)(N_s - N)$ = rotor copper loss ……..(3)

Dividing (3) by (1)

$$\frac{P_c}{P_2} = \frac{T\times\frac{2\pi}{60}(N_s - N)}{T\times\frac{2\pi}{60}\times N_s} = \frac{N_s - N}{N_s}$$

$P_c/P_2 = s$ as $(N_s - N)/N_s$ = slip s

Rotor copper loss P_c = s x Rotor input P_g

Thus total rotor copper loss is slip times the rotor input.

Now $P_2 - P_c = P_m$

$P_2 - sP_2 = P_m$

$(1 - s)P_2 = P_m$

The relationship can be expressed in the ratio from as

$$\boxed{P_2 : P_c : P_m \quad \text{is} \quad 1 : s : 1 - s}$$

This relationship is very important and very frequently required to solve the problems on the power flow diagram

Key Point : The torque produced by rotor is gross mechanical torque and due to mechanical losses entire torque can not be available to drive load. The load torque is net output torque called shaft torque or useful torque and is denoted as T_{sh}. It is related to P_{out} as,

$$\boxed{T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\left(\frac{2\pi N}{60}\right)}}$$

and $T_{sh} < T$ due to mechanical losses

Torque Equation

The torque produced in the induction motor depends on the following factors :

1. The part of rotating magnetic field which reacts with rotor and is responsible to produce induced e.m.f. in rotor.

2. The magnitude of rotor current in running condition.

3. The power factor of the rotor circuit in running condition

Combining all these factors together we get the equation of torque as-

$$T \propto \Phi I_{2r} \cos \Phi_{2r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where Φ = Flux responsible to produce induced e.m.f.

I_{2r} = Rotor current at running condition

$\cos \Phi_{2r}$ = Running p.f. of motor

The flux φ produced by the stator is proportional to stator emf E_1.

i.e $\varphi \propto E_1$ ....(2)

We know that transformation ratio K is defined as the ratio of secondary voltage (rotor voltage) to that of primary voltage (stator voltage).

$$E_2/E_1 = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Using (3) in (2) we can write,

Thus in equation (1), Φ can be replaced by E_2.

While $I_{2r} = E_{2r}/Z_{2r} = (s E_2)/\sqrt{(R_2^2 + (s X_2)^2)}$ ....(5) (E_2 – standstill rotor emf)

and $\cos \Phi_{2r} = R_2/Z_{2r} = R_2/\sqrt{(R_2^2 + (s X_2)^2)}$ ....(6)

Putting the value of flux φ, rotor current I_{2r}, power factor $\cos\theta_{2r}$ in the equation of torque we get,

$$T \propto E_2 \cdot \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\therefore \quad \boxed{T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N-m}}$$

$\therefore$ $T = (k\, s\, E_2^2\, R_2)/(R_2^2 + (s\, X_2)^2)$ ………… (7) (where k = Constant of proportionality)

The constant k is provided to be 3/2 for three phase induction motor.

$\therefore$ $k = 3/(2\, \pi\, n_s)$ ……………………..(8)

$$\boxed{T = \frac{3}{2\pi n_s} \cdot \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \text{ N - m}} \qquad \ldots (9)$$

So torque developed at any load condition can be obtained if slip at that load is known and all standstill rotor parameters are known.

Starting Torque

At start, $N_r = 0$ and slip $s = 1$. So putting $s = 1$ in the torque equation we can write expression for the starting torque T_{st} as,

$$\boxed{T_{st} = \frac{3}{2\pi n_s} \cdot \frac{E_2^2 R_2}{(R_2^2 + X_2^2)}} \qquad \ldots (10)$$

Key Point : From the equation (10), it is clear that by changing the R, starting torque can be controlled. The change in R_2 at start is possible in case of slip ring induction motor only. This is the principle used in case of slip induction motor to control the starting torque T_{st}.

Derivation of k in Torque Equation

We have seen earlier that

$$T = (k\, s\, E_2^2\, R_2)/(R_2^2 + (s\, X_2)^2)$$

and it mentioned that $k = 3/(2\pi\, n_s)$. Let us see its proof.

The rotor copper losses can be expressed as,

$$P_c = 3 \times I_{2r}^2 \times R$$

but $I_{2r} = (s\, E_2)/\sqrt{(R_2^2 + (s\, X_2)^2)}$, hence substituting above

$$P_c = 3 \times \left[\frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}} \right]^2 \times R_2$$

$$\therefore \quad P_c = \frac{3 s^2 E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Now as per **$P_2 : P_c : P_m$ is 1 : s : 1-s ,**

$P_c/P_m = s/(1-s)$

Now $P_m = T \times \omega$

$= T \times (2\pi N/60)$

$$\therefore \quad T \times \frac{2\pi N}{60} = \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \quad T = \frac{60}{2\pi N} \times \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Now $N = N_s (1-s)$ from definition of slip, substituting in above,

$$\therefore \quad T = \frac{60}{2\pi N_s (1-s)} \times \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$= \frac{3}{2\pi \left(\frac{N_s}{60} \right)} \times \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

but $N_s/60 = n_s$ in r.p.m.

So substituting in the above equation,

$$T = \frac{3}{2\pi n_s} \times \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Comparing the two torque equations we can write,

$$\boxed{k = \frac{3}{2\pi n_s}} \text{ where } n_s \text{ is in r.p.s.}$$

Maximum Torque Condition

$$T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi n_s}$$

If the impedance of the stator winding is assumed to be negligible and supply voltage V is kept constant, then flux ϕ and E_2 both remains constant. Hence

$$T = KsE_2^2 \frac{R_2}{R_2^2 + (sX_2)^2}$$

So, for torque to be maximum

$$\frac{dT}{ds} = 0$$

On differentiating and after putting the terms equal to zero we get

$$s^2 = \frac{R_2^2}{X_2^2}$$

So, when slip $s = R_2 / X_2$, the torque will be maximum and this slip is called maximum slip S_m and it is defined as the ratio of rotor resistance to that of rotor reactance.

NOTE : At starting S = 1, so the maximum starting torque occur when rotor resistance is equal to rotor reactance. $R_2 = X_2$

Equation of Maximum Torque

The torque will be maximum when slip $s = R_2 / X_2$

Substituting the value of this slip in torqur equation we get the maximum value of torque as,

$$T_{max} = K\frac{E_2^2}{2X_2}\ N - m$$

CONCLUSION : From the above equation it is concluded that

1. The maximum torque is directly proportional to square of rotor induced emf at the standstill.
2. The maximum torque is inversely proportional to rotor reactance.
3. The maximum torque is independent of rotor resistance.

4. The slip at which maximum torque occur depends upon rotor resistance, R_2. So, by varying the rotor resistance, maximum torque can be obtained at any required slip.

Torque Ratios

The performance of the motor is sometimes expressed in terms of comparison of various torques such as full load torque, starting torque and maximum torque. The comparison is obtained by finding out ratios of these torques.

1. Full load and Maximum Torque Ratio

In general, $T \propto (s E_2^2 R_2)/(R_2^2 + (s X_2)^2)$

Let s_f = Full load slip

∴ $T_{F.L.} \propto (s_f E_2^2 R_2)/(R_2^2 + (s_f X_2)^2)$

and s_m = Slip for maximum torque T_m

∴ $T_m \propto (s_m E_2^2 R_2)/(R_2^2 + (s_f X_2)^2)$

$$\therefore \quad \frac{T_{F.L.}}{T_m} = \frac{s_f E_2^2 R_2}{[R_2^2 + (s_f X_2)^2]} \times \frac{[R_2^2 + (s_m X_2)^2]}{s_m E_2^2 R_2}$$

$$\therefore \quad \frac{T_{F.L.}}{T_m} = \frac{s_f}{s_m} \times \frac{[R_2^2 + (s_m X_2)^2]}{[R_2^2 + (s_f X_2)^2]}$$

Dividing both numerator and denominator by X_2^2 we get,

$$\therefore \quad \frac{T_{F.L.}}{T_m} = \frac{s_f}{s_m} \times \frac{\left[\frac{R_2^2}{X_2^2} + s_m^2\right]}{\left[\frac{R_2^2}{X_2^2} + s_f^2\right]}$$

But $R_2/X_2 = s_m$

$T_{F.L.}/T_m = (s_f \text{ x } 2\, s_m^2)/(s_m \text{ x } (s_m^2 + s_f^2))$

$T_{F.L.}/T_m = (2\, s_f\, s_m)/(s_m^2 + s_f^2)$

2.Starting Torque and Maximum Torque Ratio

Against starting with torque equation as,

$T \propto (s E_2^2 R_2)/(R_2^2 + (s X_2)^2)$

Now for T_{st}, $s = 1$

$T_{st} \propto (E_2^2 R_2)/(R_2^2 + (X_2)^2)$

While for T_m, $s = s_m$

$$\therefore \quad T_m \propto \frac{s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2}$$

$$\therefore \quad \frac{T_{st}}{T_m} = \frac{E_2^2 R_2}{[R_2^2 + X_2^2]} \times \frac{[R_2^2 + (s_m X_2)^2]}{s_m E_2^2 R_2}$$

$$\therefore \quad \frac{T_{st}}{T_m} = \frac{[R_2^2 + (s_m X_2)^2]}{s_m [R_2^2 + X_2^2]}$$

Dividing both numerator and denominator by X_2^2 we get,

$$\therefore \quad \frac{T_{st}}{T_m} = \frac{\left[\frac{R_2^2}{X_2^2} + s_m^2\right]}{s_m \left[\frac{R_2^2}{X_2^2} + 1\right]}$$

Substituting $R_2/X_2 = s_m$

$$\therefore \quad \boxed{\frac{T_{st}}{T_m} = \frac{2 s_m^2}{s_m (1 + s_m^2)} = \frac{2 s_m}{1 + s_m^2}}$$

Infact using the same method, ratio of any two torques at two different slip values can be obtained.

Sometimes using the relation, $R_2 = a X_2$ the torque ratios are expressed interms of constant a as,

$T_{F.L.}/T_m = (a s_f)/(a^2 + s_f^2)$

and $T_{st}/T_m = 2a/(1 + a^2)$

where $a = R_2/X_2 = s_m$

Torque-Slip Characteristics

As the induction motor is located from no load to full load, its speed decreases hence slip increases. Due to the increased. load, motor has to produce more torque to satisfy load demand. The torque ultimately depends on slip.

for a constant supply voltage, E_2 is also constant. So we can write torque equations as,

$$\boxed{T \propto \frac{s R_2}{R_2^2 + (s X_2)^2}}$$

Now to judge the nature of torque-slip characteristics let us divide the slip range (s = 0 to s = 1) into two parts and analyse them independently.

i) Low slip region :

In low slip region, 's' is very very small. Due to this, the term $(s\ X_2)^2$ is so small as compared to R_2^2 that it can be neglected.

$$\therefore \quad \boxed{T \propto \frac{s R_2}{R_2^{\,2}} \propto s} \qquad \text{As } R_2 \text{ is constant.}$$

Hence in low slip region torque is directly proportional to slip. So as load increases, speed decreases, increasing the slip. This increases the torque which satisfies the load demand.

Hence the graph is straight line in nature.

At $N = N_s$, $s = 0$ hence $T = 0$. As no torque is generated at $N = N_s$, motor stops if it tries to achieve the synchronous speed. Torque increases linearly in this region, of low slip values

ii) High slip region :

In this region, slip is high i.e. slip value is approaching to 1. Here it can be assumed that the term R_2^2 is very very small as compared to $(s\ X_2)^2$. Hence neglecting from the denominator, we get

$$\boxed{T \propto \frac{s R_2}{(sX_2)^2} \propto \frac{1}{s}} \text{ where } R_2 \text{ and } X_2 \text{ are constants}$$

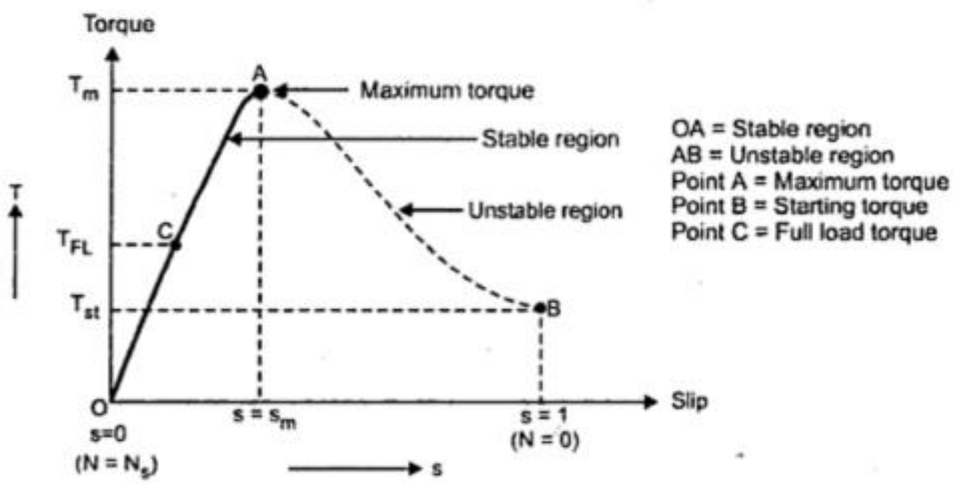
So in high slip region torque is inversely proportional to the slip. Hence its nature is like rectangular hyperbola.

Now when load increases, load demand increases but speed decreases. As speed decreases, slip increases. In high slip region as $T\ \alpha 1/s$, torque decreases as slip increases.

But torque must increases to satisfy the load demand. As torque decreases, due to extra loading effect, speed further decreases and slip further increases. Again torque decreases as $T\ \alpha 1/s$ hence same load acts as an extra load due to reduction in torque produced. Hence speed further drops. Eventually motor comes to standstill condition. The motor can not continue to rotate at any point in this high slip region. Hence this region is called unstable region of operation.

So torque-slip characteristics has two parts,

1. Straight line called stable region of operation
2. Rectangular hyperbola called unstable region of operation



Now the obvious question is upto which value of slip, torque - slip characteristics represents stable operation ?

In low slip region, as load increases, slip increases and torque also increases linearly. Every motor has its own limit to produce a torque. The maximum torque, the motor can produces as load increases is T_m which occurs at $s = s_m$. So linear behaviour continues till $s = s_m$.
maximum torque which motor can produce is also called **breakdown torque or pull out torque**

Range $s = 0$ to $s = s_m$ is called low slip region, known as stable region of operation. Motor always operates at a point in this region. And range $s = s_m$ to $s = 1$ is called high slip region which is rectangular hyperbola, called unstable region of operation. Motor can not continue to rotate at any point in this region.
At s = 1, N = 0 i.e. start, motor produces a torque called starting torque denoted as T_{st}.

Effect of Change in Rotor Resistance on Torque

It is shown that in slip ring induction motor, externally resistance can be added in the rotor. Let us see the effect of change in rotor resistance on the torque produced.

Let R_2 = Rotor resistance per phase

Corresponding torque, $T \alpha (s E_2^2 R_2)/\surd(R_2^2 + (s X_2)^2)$

Now externally resistance is added in each phase of rotor through slip rings.

Let R_2' = New rotor resistance per phase

Corresponding torque $T' \alpha (s E_2^2 R_2')/\surd(R_2'^2 + (s X_2)^2)$

Similarly the starting torque at s = 1 for R_2 and R_2' can be written as

$$T_{st} \alpha (E_2^2 R_2)/\surd(R_2^2 + (X_2)^2)$$

and $$T'st \alpha (E_2^2 R'_2)/\surd(R'_2{}^2 + (X_2)^2)$$

Maximum torque $$T_m \alpha (E_2^2)/(2X_2)$$

Key Point : It can be observed that T_m is independent of R_2 hence whatever may be the rotor resistance, maximum torque produced never change but the slip and speed at which it occurs depends on R_2.

For R_2, $s_m = R_2/X_2$ where T_m occurs

For R_2', $s_m' = R_2'/X_2'$ where same T_m occurs

As $R_2' > R_2$, the slip $s_m' > s_m$. Due to this, we get a new torque-slip characteristics for rotor resistance . This new characteristics is parallel to the characteristics for with same but T_m occurring at s_m'. The effect of change in rotor resistance on torque-slip characteristics shown in the Fig. 1.

It can be seen that the starting torque T'st for R_2' is more than Tst for R_2. Thus by changing rotor resistance the starting torque can be controlled.

If now resistance is further added to rotor to get resistance as R_2' and so on, it can be seen that T_m remains same but slip at which it occurs increases to s_m' and so on. Similarly starting torque also increases to T'st and so on.

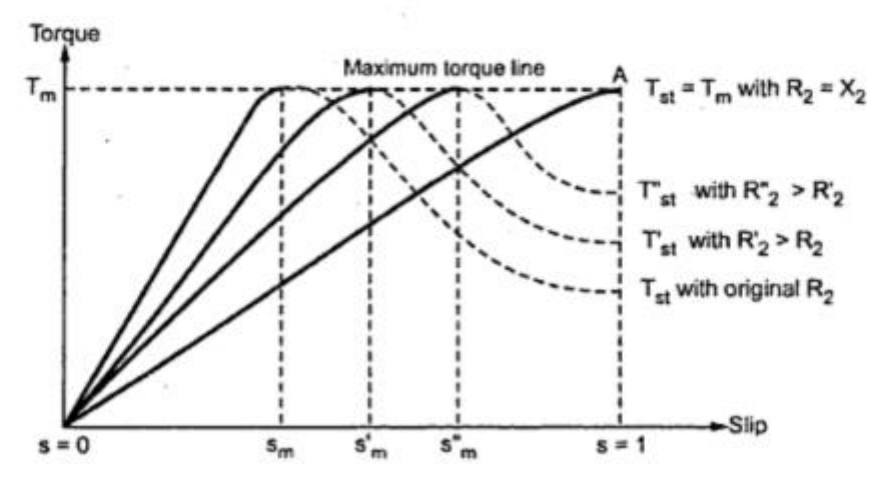


Fig. 1 Effect of rotor resistance on torque-slip curves

If maximum torque Tm is required at start then $s_m = 1$ as at start slip is always unity, so

$$s_m = R_2/X_2 = 1$$

$R_2 = X_2$ Condition for getting $Tst = T_m$

Key Point : Thus by adding external resistance to rotor till it becomes equal to X_2, the maximum torque can be achieved at start.

It is represented by point A in the Fig. 1.

If such high resistance is kept permanently in the circuit, there will be large copper losses ($I^2 R$) and hence efficiency of the motor will be very poor. Hence such added resistance is cut-off gradually and finally removed from the rotor circuit, in the normal running condition of the motor. So this method is used in practice to achieve higher starting torque hence resistance in rotor is added only at start.

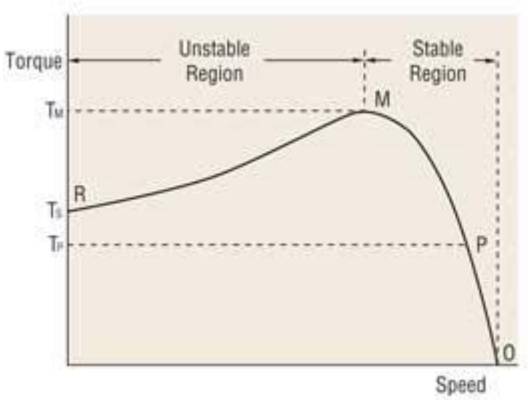
Thus good performance at start and in the running condition is ensured.

Key Point : This is possible only in case of slip type of induction motor as in squirrel cage due to short circuited rotor, extra rotor resistance can not be added.

Torque-Speed Characteristics

At N = 0, the starting condition, motor produces a torque called starting torque.

At N = T_s, the motor stops as it can not produce any torque, as induction motor can not rotate as synchronous motor.



For low slip region, i.e. speeds near the region is stable and the characteristics is straight in nature. Fall in speed from no load to full load is about 4 to 6 %. It can be seen that for the stable region of operation, the characteristics is similar to that of d.c. shunt motor. Due to this, three phase induction motor is practically said to be 'constant speed' motor as drop in speed from no load to full load is not significant.

Losses in Induction Motor

There are two types of losses occur in three phase induction motor. These losses are,

1. Constant or fixed losses,
2. Variable losses.

i) Constant losses :

These can be further classified as core losses and mechanical losses.

Core losses occur in stator core and rotor core. These are also called iron losses. These losses include eddy current losses and hysteresis losses. The eddy current losses are minimised by using laminated construction while hysteresis losses are minimised by selecting high grade silicon steel as the material for stator and rotor.

The iron losses depends on the frequency. The stator frequency is always supply frequency hence stator iron losses are dominate. As against this in rotor circuit, the frequency is very small which is slip times the supply frequency. Hence rotor iron losses are very small and hence generally neglected, in the running condition.

The mechanical losses include frictional losses at the bearings and windings losses. The friction changes with speed but practically the drop in speed is very small hence these losses are assumed to be the part of constant losses.

ii) Variable losses :

This include the copper losses in stator and rotor winding due to current flowing in the winding. As current changes as load changes as load changes, these losses are said to be variable losses.

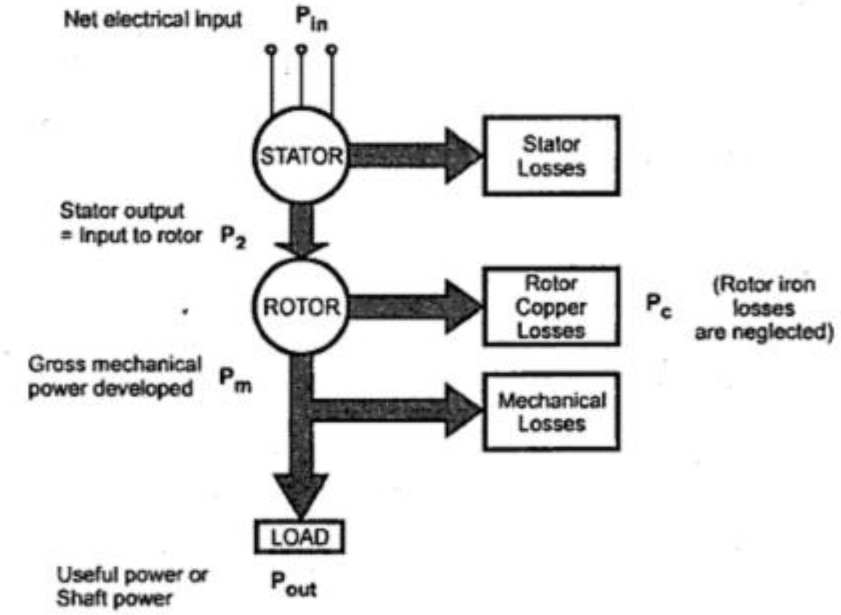
Generally stator iron losses are combined with stator copper losses at a particular load to specify total stator losses at particular load condition.

Rotor copper loss = 3 $I_{2r}^2 R_2$ ......Analysed separately

where I_{2r} = Rotor current per phase at a particular load

R_2 = Rotor resistance per phase

Power Flow in an Induction Motor



$$\boxed{P_{in} = \sqrt{3}\ V_L\ I_L \cos \phi}$$

The part of this power is utilised to supply the losses in the stator which are stator core as well as copper losses.

So $\boxed{P_2 = P_{in} - \text{stator losses (core + copper)}}$

rotor losses are rotor copper losses denoted as P_c.

So $\boxed{P_c = 3 \times I_{2r}^2 \times R_2}$

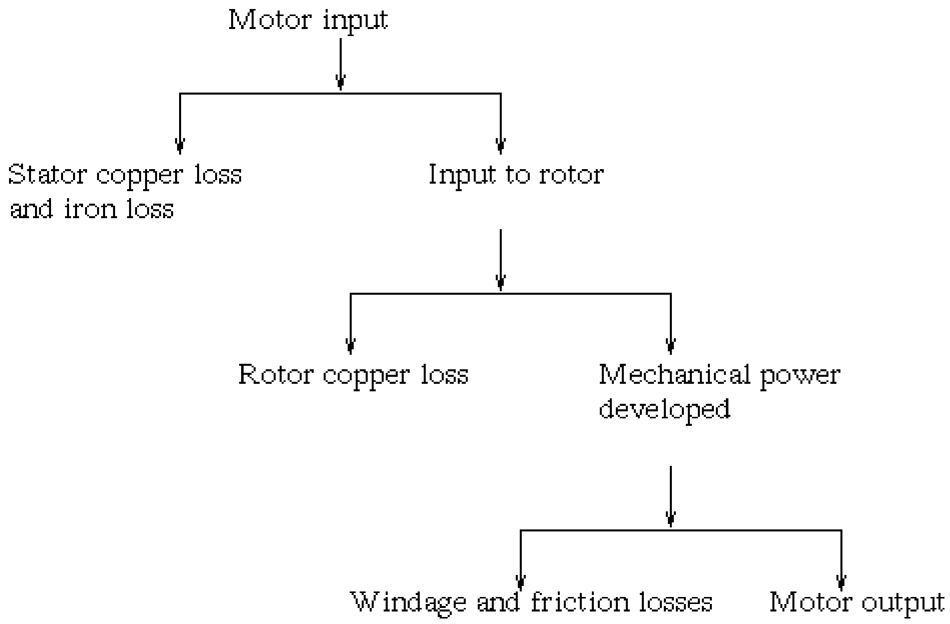
gross mechanical power developed by the motor

$\therefore$ $\boxed{P_m = P_2 - P_c}$

Some part of P_m is utilised to provide mechanical losses like friction and windage. So net output power of the motor denoted as P_{out}. This is also called shaft power.

$\therefore$ $\boxed{P_{out} = P_m - \text{Mechanical losses.}}$

The rating of the motor is specified in terms of value of P_{out} when load condition is full load condition.



From the power flow diagram we can define,

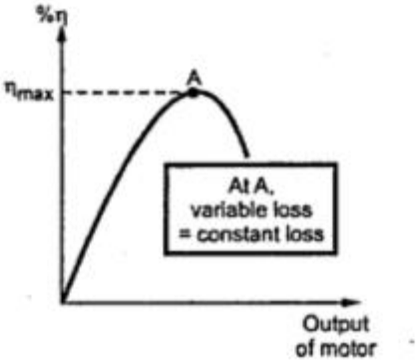
$$\text{Rotor efficiency} = \frac{\text{rotor output}}{\text{rotor input}} = \frac{\text{gross mechanical power developed}}{\text{rotor input}}$$

$$= P_m / P_2$$

$$\text{Net motor efficiency} = \frac{\text{net output at shaft}}{\text{net electrical input to motor}} = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{P_{out}}{P_{in}}$$

The maximum efficiency occurs when variable losses becomes equal to constant losses. When motor is on no load, current drawn by the motor is small. Hence efficiency is low. As load increases, current increases so copper losses also increases. When such variable losses achieve the same value as that of constant losses, efficiency attains its maximum value. If load is increased further, variable losses becomes greater than constant losses hence deviating from condition for maximum, efficiency starts decreasing.



Efficiency curve for an induction motor

Induction Motor as a Transformer

We know that, transformer is a device in which two windings are magnetically coupled and when one winding is excited by a.c. supply of certain frequency, the e.m.f. gets induced in the second winding having same frequency as that of supply given to the first winding. The winding to which supply is given is called primary winding while winding in which e.m.f. gets induced is called secondary winding. The induction motor can be regarded as the transformer.

The difference is that the normal transformer is an alternating flux transformer while induction motor is rotating flux transformer. The normal transformer has no air gap as against this an induction motor has distinct air gap between its stator and rotor.

In an alternating flux transformer the frequency of induced e.m.f. and current in primary and secondary is always same. However in the induction motor frequency of e.m.f. and current on the stator side remains same but frequency of rotor e.m.f. and current depends on the slip and slip depends on load on the motor. So we have a variable frequency on the rotor side. But it is important to remember that at start when N = 0 the value of slip is unity (s = 1), then frequency of supply to the stator and of induced e.m.f. in the rotor is same. The effect of slip on the rotor parameters is already discussed in the previous section.

And last difference is that in case of the alternating flux transformer the entire energy present in the secondary circuit, is in the electrical form. As against this, in an induction motor part of its energy in the rotor circuit is in electrical form and the remaining part is converted into mechanical form.

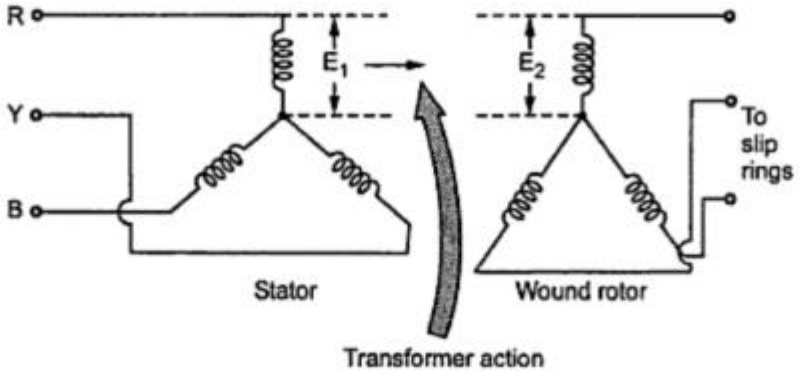


Fig. 1 Induction motor as a transformer

In general, an induction motor can be treated as a generalised transformer as shown in the Fig. 1. In this, the slip ring induction motor with star connected stator and rotor is shown.

So if E_1 = Stator e.m.f. per phase in volts.

E_2 = Rotor induced e.m.f. per phase in volts at start when motor is at standstill.

Then according to general transformer there exists a fixed relation between E_1 and E_2 called transformer ratio.

$\therefore$ At start when N =0 , s =1

and we get,

$$\boxed{\frac{E_2}{E_1} = K = \frac{\text{Rotor turns / phase}}{\text{Stator turns / phase}}}$$

Key Point : So if stator supply voltage is known and ratio of stator to rotor turns per phase is known then the rotor induced e.m.f. on standstill can be obtained.

Equivalent Circuit of Induction Motor

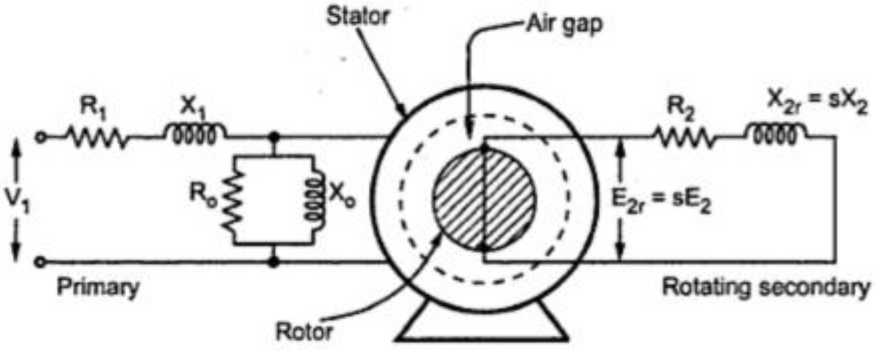
The energy transfer from stator to rotor of the induction motor takes place entirely with the help of a flux mutually linking the two. Thus stator acts as a primary while the rotor acts as a rotating secondary when induction motor is treated as a tranformer.

If E_1 = Induced voltage in stator per phase
E_2 = Rotor induced e.m.f. per phase on standstill
k = Rotor turns / Stator turns
then $k = E_2/ E_1$

Thus if V_1 is the supply voltage per phase to stator, it produces the flux which links with both stator and rotor. Due to self induction E_1, is the induced e.m.f. in stator per phase while E_2 is the induced e.m.f. in rotor due to mutual induction, at standstill. In running condition the induced e.m.f. in rotor becomes E_{2r} which is s E_2.

Now E_{2r} = Rotor induced e.m.f. in running condition per phase
R_2 = Rotor resistance per phase
X_{2r} = Rotor reactance per phase in running condition
R_1 = Stator resistance per phase
X_1 = Stator reactance per phase

So induction motor can be represented as a transformer as shown in the Fig.



Induction motor as a transformer

When induction motor is on no load, it draws a current from the supply to produce the flux in air gap and to supply iron losses.

1. I_c = Active component which supplies no load losses
2. I_m = Magnetizing component which sets up flux in core and air gap

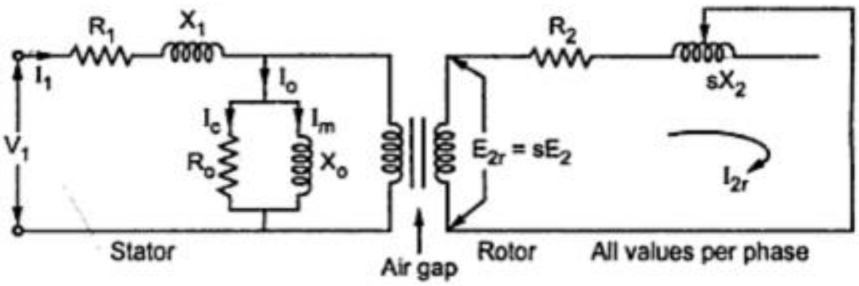
These two currents give us the elements of an exciting branch as,

R_o = Representing no load losses = V_1 / I_c

and X_o = Representing flux set up = V_1 / I_m

Thus, $\bar{I}_o = \bar{I}_c + \bar{I}_m$

The equivalent circuit of induction motor thus can be represented as shown in the Fig.



Basic equivalent circuit

The stator and rotor sides are shown separated by an air gap.

I_{2r} = Rotor current in running condition

$= E_{2r} / Z_{2r} = (s E_2)/\sqrt{(R_2^2 + (s X_2)^2)}$

It is important to note that as load on the motor changes, the motor speed changes. Thus slip changes. As slip changes the reactance X_{2r} changes. Hence $X_{2r} = sX_2$ is shown variable.

Representing of rotor impedance :

It is shown that, $I_{2r} = (sE_2)/\sqrt{(R_2^2 + (s X_2)^2)} = E_2 / \sqrt{((R_2/s)^2 + X_2^2)}$

So it can be assumed that equivalent rotor circuit in the running condition has fixed reactance X_2, fixed voltage E_2 but a variable resistance R_2/s, as indicated in the above equation.

Now $R_2/s = R_2 + (R_2/s) - R_2$

$$\therefore \quad \boxed{\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s}-1\right) = R_2 + R_2\left(\frac{1-s}{s}\right)}$$

So the variable rotor resistance R_2/s has two parts.

1. Rotor resistance R_2 itself which represents copper loss.

2. $R_2(1 - s)/s$ which represents load resistance R_L. So it is electrical equivalent of mechanical load on the motor.

Key Point: Thus the mechanical load on the motor is represented by the pure resistance of value $R_2(1 -s)/s$.

So rotor equivalent circuit can be shown as,

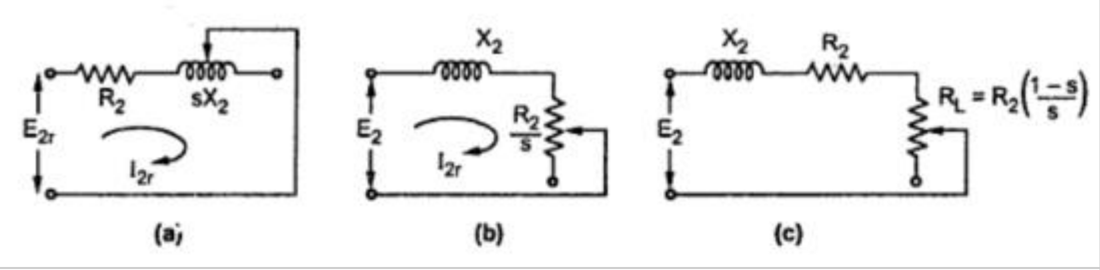


Fig. 3 Rotor equivalent circuit

Now let us obtain equivalent circuit referred to stator side.

Equivalent circuit referred to stator :

Transfer all the rotor parameters to stator,

$k = E_2/E_1$ = Transformation ratio

$E_2' = E_2/ k$

The rotor current has its reflected component on the stator side which is I_{2r}'.

$I_{2r}' = k I_{2r} = (k s E_2)/\sqrt{(R_2^2 +(s X_2)^2)}$

$X_2' = X_2/K^2$ = Reflected rotor reactance

$R_2' = R_2/K^2$ = Reflected rotor resistance

$R_L' = R_L/K^2 = (R_2/K^2)(1-s / s)$

$= R_2' (1-s / s)$

Thus R_L' is reflected mechanical load on stator.

So equivalent circuit referred to stator can be shown as in the Fig. 4

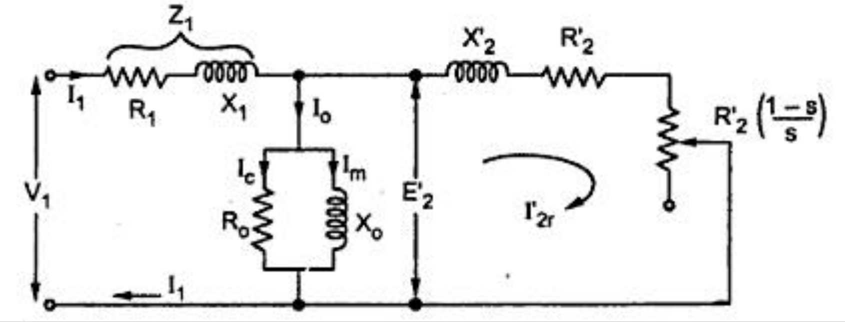


Fig. 4 Equivalent circuit referred to stator

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The resistance $R_2' (1 -s)/ s = R_L'$ is fictitious resistance representing the mechanical load on the motor.

Approximate Equivalent Circuit

Similar to the transformer the equivalent circuit can be modified by shifting the exciting current (R_o and X_o) purely across the supply, to the left of R_1 and X_1. Due to this, we are neglecting the drop across R_1 and X_1 due to I_o, which is very small. Hence the circuit is called approximate equivalent circuit. The circuit is shown in the Fig.5.

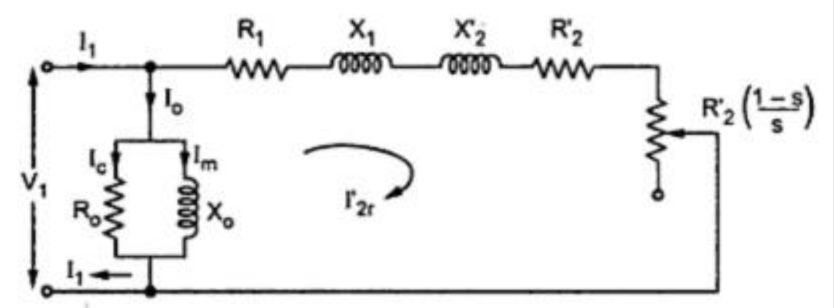

Fig. 5 Approximate equivalent circuit

Now the resistance R_1 and R_2' while reactance X_1 and X_2' can be combined. So we get,

R_{1e} = Equivalent resistance referred to stator = $R_1 + R_2'$

X_{1e} = Equivalent reactance referred to stator = $X_1 + X_2'$

$R_{1e} = R_1 + (R_2/K^2)$

and $X_{1e} = X_1 + (X_2/K^2)$

While $\bar{I}_1 = \bar{I}_o + \bar{I}_{2r}'$ ....phasor diagram

and $\bar{I}_o = \bar{I}_c + \bar{I}_m$

Thus the equivalent circuit can be shown in the Fig.6.

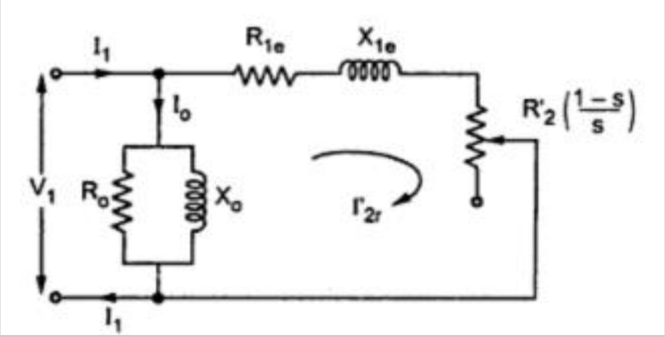

Fig. 6

Power Equations from Equivalent Circuit

With reference to approximate equivalent circuit shown in the Fig. 6, we can write various power equations as,

P_{in} = input power = $3\, V_1\, I_1 \cos \Phi$

where V_1 = Stator voltage per phase

I_1 = Current drawn by stator per phase

$\cos\Phi$ = Power factor of stator

Stator core loss = $I_m^2 R_o$

Stator copper loss = $3 I_1^2 R_o$

where R_1 = Stator resistance per phase

P_2 = Rotor input = $(3 I_{2r}'^2 R_2')/s$

P_c = Rotor copper loss = $3 I_{2r}'^2 R_2'$

Thus $P_c = s P_2$

P_m = Gross mechanical power developed

$$\therefore \quad P_m = P_2 - P_c = \frac{3(I_{2r}')^2 R_2'}{s} - 3(I_{2r}')^2 R_2' = 3(I_{2r}')^2 R_2' \left(\frac{1-s}{s}\right)$$

T = Torque developed

$$\therefore \quad T = \frac{P_m}{\omega} = \frac{3(I_{2r}')^2 R_2' \left(\frac{1-s}{s}\right)}{\frac{2\pi N}{60}}$$

where N = Speed of motor

But $N = N_s (1-s)$ =, so substituting in above

$$T = \frac{\frac{3(I_{2r}')^2 R_2'}{s}}{\frac{2\pi N_s}{60}} = 9.55 \times \frac{\frac{3(I_{2r}')^2 R_2'}{s}}{N_s} \text{ N-m}$$

and $I_{2r}' = V_1/((R_{1e} + R_L') + j X_{1e})$

where $R_L' = R_2' (1-s)/s$

$I_{2r}' = V_1/\sqrt{((R_{1e} + R_L')^2 + X_{1e}^2)}$

Key Point : Remember that in all the above formula all the values per phase values.

Maximum Power Output

Consider the approximate equivalent circuit as shown in the Fig.7

In this circuit, the exciting current I_o is neglected hence the exciting no load branch is not shown.

$\therefore \quad I_1 = I_{2r}'$

The total impedance is given by,

$Z_T = (R_{1e} + R_L')+$ where $R_L' = R_2' (1-s)/s$

$$I_1 = V_1 / \sqrt{(R_{1e} + R_L')^2 + (X_{1e})^2}$$

The power supplied to the load i.e. P_{out} per phase is,

Per phase $P_{out} = I_1^2\ R_L'$ watts per phase

$\therefore$ Total $= 3\ I_1^2\ R_L'$

$$\therefore \quad \boxed{P_{out} = 3\,\frac{V_1^2}{[(R_{1e}+R_L')^2+(X_{1e})^2]}\,(R_L')}$$

To obtain maximum output power, differentiate the equation of total P_{out} with respect to variable R_L' and equal to zero.

$$\therefore \quad \frac{d}{dR_L'}\left[\frac{3\,V_1^2(R_L')}{[(R_{1e}+R_L')^2+(X_{1e})^2]}\right] = 0$$

$$\therefore \quad [(R_{1e}+R_L')^2+(X_{1e})^2]\,[3\,V_1^2] - 3\,V_1^2(R_L')\,[2\,(R_{1e}+R_L')] = 0$$

$$\therefore \quad (R_{1e}+R_L')^2+(X_{1e})^2 - 2\,(R_L')\,(R_{1e}+R_L') = 0 \qquad \text{... Taking } 3\,V_1^2 \text{ common}$$

$$\therefore \quad R_{1e}^2 + (R_L')^2 + 2\,R_{1e}\,R_L' + X_{1e}^2 - 2\,R_{1e}\,R_L' - 2(R_L')^2 = 0$$

$$\therefore \quad R_{1e}^2 + X_{1e}^2 = (R_L')^2$$

But $Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2}$ = Leakage impedance referred to stator

$\therefore \quad Z_{1e}^2 = R_L'^2$

Thus the mechanical load on the induction motor should be such that the equivalent load resistance referred to stator is equal to the total leakage impedance of motor referred to stator.

Slip at maximum P_{out} : This can be obtained as,

$R_L' = Z_{1e} = R_2'(1-s)/s$ where $R_L' = R_2/K^2$

$\therefore \quad s\,Z_{1e} = R_2' - sR_2'$

$\therefore \quad s(Z_{1e} + R_2') = R_2'$

$$\boxed{s = \frac{R_2'}{(R_2' + Z_{1e})}}$$

This is slip at maximum output.

Expression for maximum P_{out} : Using the condition obtained in expression of total P_{out}, we can get maximum P_{out}.

$\therefore \quad (P_{out})_{max} = 3\, I_1^2\, Z_{1e} \qquad \text{as } R_L' = Z_{1e}$

$$= 3\, \frac{V_1^2}{(R_{1e}+Z_{1e})^2 + (X_{1e})^2} \cdot Z_{1e} \quad \text{as} \quad R_L' = Z_{1e}$$

$$= 3\, \frac{V_1^2}{(R_{1e}^2 + 2\,R_{1e}\,Z_{1e} + Z_{1e}^2 + X_{1e}^2)} \cdot Z_{1e}$$

But $\quad R_{1e}^2 + X_{1e}^2 = Z_{1e}$

$$\therefore \quad (P_{out})_{max} = 3\, \frac{V_1^2}{2Z_{1e}^2 + 2R_{1e}\,Z_{1e}} \cdot Z_{1e} = 3\, \frac{V_1^2}{2\,Z_{1e} + (R_{1e}+Z_{1e})} \cdot Z_{1e}$$

$$\therefore \quad \boxed{(P_{out})_{max} = \frac{3\,V_1^2}{2\,(R_{1e}+Z_{1e})} \text{ watts}}$$

Maximum Torque

In case of induction motor, the speed of the motor decreases with increase in load. Thus the maximum power output is not obtained at a slip which corresponds to maximum torque. In the previous section we have seen the condition for maximum power output. In this section we will find the condition which gives maximum torque.

The expression for torque is given by,

$$T = \frac{3(I_{2r}')^2 R_2'}{s\,\omega_s} = \frac{3(I_{2r}')^2 R_2'}{s\left(\frac{2\pi N_s}{60}\right)}$$

The condition for maximum torque can be obtained from maximum power transfer theorem. When $I_{2r}'^2\, R_2'/s$ is maximum consider the approximate equivalent circuit of induction motor as shown in The Fig. 8.

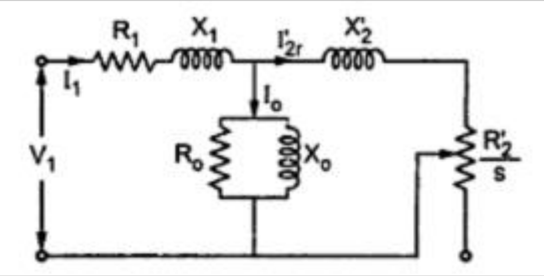

Fig. 8

The value of R_o is assumed to be negligible. Hence the circuit will be reduced as shown below.

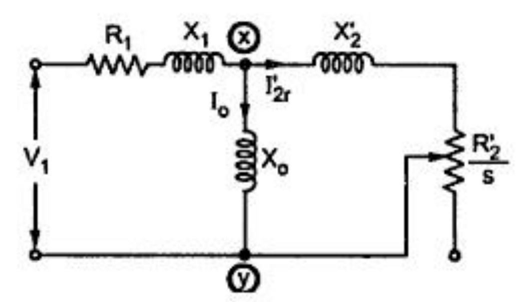

Fig. 9

The thevenin's equivalent circuit for the above network is shown in the Fig.10 across the terminals x and y.

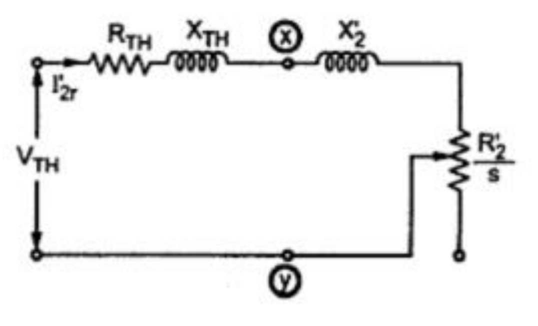

Fig. 10

Now,

$$Z_{TH} = R_{TH} + jX_{TH} = (R_1 + j\,X_1) \parallel j\,X_o$$

$$V_{TH} = \frac{V_1(jX_o)}{R_1 + j(X_{TH} + X_o)}$$

The mechanical torque developed by rotor is maximum if there is maximum power transfer to the resistor R_2'/s. This takes place when R_2'/s equals to impedance looking back into the supply source.

$$\frac{R'_2}{s} = R_{TH} + j(X_{TH} + X'_2)$$

$$\frac{R'_2}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}$$

$$\therefore \qquad s = s_m = \frac{R'_2}{\sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}}$$

This is the slip corresponding to the maximum torque. The maximum torque is given by,

$$T_m = \frac{3}{\omega_s}\cdot\frac{R'_2}{s_m}\cdot(I'_{2r})^2$$

$$\frac{R'_2}{s_m} = \sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}$$

$$\therefore \quad I'_{2r} = \frac{V_{TH}}{\sqrt{\left(R_{TH}+\frac{R'_2}{s_m}\right)^2+(X_{TH}+X'_2)^2}}$$

$$I'^2_{2r} = \frac{V^2_{TH}}{\left(R_{TH}+\frac{R'_2}{s_m}\right)^2+(X_{TH}+X'_2)^2}$$

Substituting,

$$T_m = \frac{3}{\omega_s}\cdot\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\cdot\frac{V^2_{TH}}{\left[R_{TH}+\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\right]^2+(X_{TH}+X'_2)^2}$$

$$= \frac{3}{\omega_s}\cdot\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\cdot\frac{V^2_{TH}}{2R^2_{TH}+2R_{TH}\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}+2(X_{TH}+X'_2)^2}$$

$$= \frac{3}{\omega_s}\cdot\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\cdot\frac{R^2_{TH}}{2\left[R^2_{TH}+(X_{TH}+X'_2)^2+R_{TH}\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\right]}$$

$$= \frac{3}{\omega_s}\cdot\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\cdot\frac{0.5\,V^2_{TH}}{\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\left[R_{TH}+\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}\right]}$$

$$\therefore \quad \boxed{T_m = \frac{3}{\omega_s}\cdot\frac{0.5\,V^2_{TH}}{R_{TH}+\sqrt{R^2_{TH}+(X_{TH}+X'_2)^2}}}$$

From the above expression, it can be seen that the maximum torque is independent of rotor resistance.

Synchronous Watt

The torque produced in the induction motor is given by,

$$T = \frac{\frac{3(I'_{2r})^2 R'_2}{s}}{\frac{2\pi N_s}{60}} = \frac{P_2}{\frac{2\pi N_s}{60}} \text{ N-m}$$

Thus torque is directly proportional to the rotor input. By defining new unit of torque which is synchronous watt we can write,

$T = P_2$ synchronous-watts

If torque is given in synchronous-watts then it can be obtained in N-m as,

$$\boxed{1 \text{ syn-watt} = \frac{60}{2\pi N_s} \text{ N-m}}$$

i.e. $$\boxed{1 \text{ N-m} = \frac{2\pi N_s}{60} \text{ syn-watt}}$$

Key Point : Unit synchronous watt can be defined as the torque developed by the motor such that the power input to the rotor across the air gap is 1 W while running at synchronous speed.

Phasor Diagram of Induction Motor

The phasor diagram of loaded induction motor is similar to the loaded transformer. The only difference is the secondary of induction motor is rotating and short circuited while transformer secondary is stationary and connected to load.

Let Φ = Magnetic flux links with both primary and secondary.

There is self induced e.m.f. E_1 in the stator while a mutually induced e.m.f. E_{2r} in the rotor.

Let R_1 = Stator resistance per phase.

X_1 = Stator reactance per phase

The stator voltage per phase V_1 has to counter balance self induced e.m.f. $E_1$and has to supply voltage drops I_1 R_1 and I_1 X_1. So on stator side we can write,

$$\overline{V_1} = -\overline{E_1} + \overline{I_1 R_1} + j\,\overline{I_1 X_1} = \overline{E_1} + \overline{I_1}\,(R_1 + j\,X_1) = -\overline{E}_1 + \overline{I}_1\overline{Z}_1$$

The rotor induced e.m.f. in the running condition has to supply the drop across impedances as rotor short circuited.

$$\therefore \qquad \overline{E}_{2r} = \overline{I_{2r} R_2} + j\overline{I_{2r} X_2} = \bar{I}_{2r}(R_2 + jX_2) = \overline{I_{2r}}\ \overline{Z_{2r}}$$

The value of E_{2r} depends on the ratio of rotor turns to stator turns.

The rotor current in the running condition is I_{2r} which lags E_{2r} by rotor p.f. angle Φ_{2r}.

The reflected rotor current I_{2r}' on stator side is the effect of load and is given by,

$$I_{2r}' = K\, I_{2r}$$

The induction motor draws no load current I_o which is phasor sum of I_c and I_m. The total stator current drawn from supply is,

$$\bar{I}_1 = \bar{I}_o + \bar{I}_{2r}'$$

The Φ_1 is angle between V_1 and I_1 and cos Φ_1 gives the power factor of the induction motor.

Thus using all above relations the phasor diagram of induction motor on load can be obtained.

The steps to draw phasor diagram are,

1. Takes Φ as reference phasor.
2. The induced voltage E_1 lags Φ by 90^o.
3. Show - E_1 by reversing voltage phasor.
4. The phasor E_{2r} is in phase with E_1. So I_{2r} show lagging E_{2r} i.e. E_1 direction by Φ_{2r}.
5. Show I_{2r} R_2 in phase with I_{2r} and I_{2r} X_{2r} leading the resistive drop by 90^o, to get exact location of.
6. Reverse I_{2r} to get I_{2r}'.
7. I_m is in phase with Φ while Ic is at leading with. Add I_m and Ic to get I_o.
8. Add I_o and I_{2r}' to get I_1.
9. From tip of - E_1 phasor, add I_1 R_1 in phase with I_1 and I_1X_1 at 90^o leading to I_1 to V_1 get phasor.
10. Angle between V_1 and I_1 is Φ_1.

The phasor diagram is shown in the Fig.

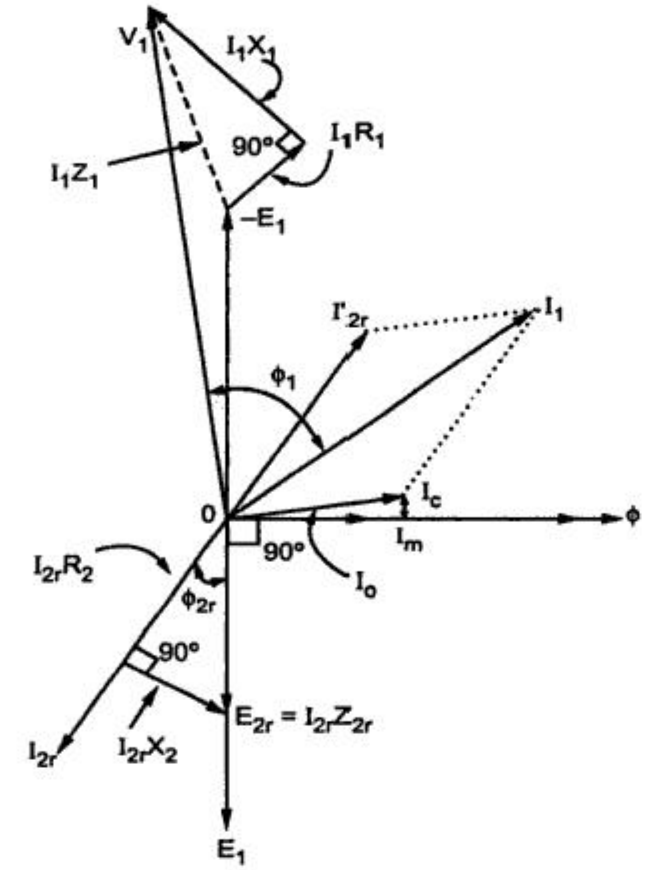

Fig.On load phasor diagram of induction motor

Effect of harmonics on Performance of 3-ph Induction Motor

The induction motor performance is affected by the harmonics in the time variation of the impressed voltage. But its effect on the performance of the motor is not predominant hence it is not considered here.

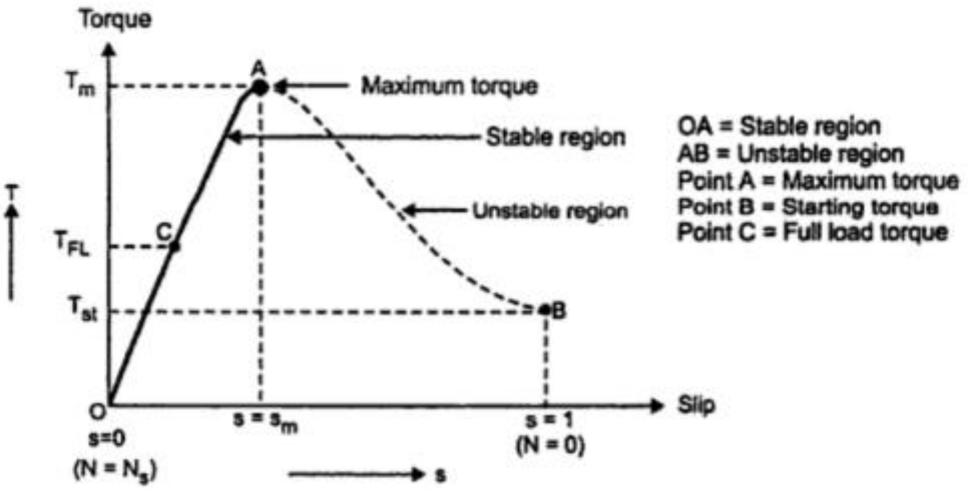

Fig. 1 Torque speed characteristics

The torque-slip characteristics as shown in Fig.1 is obtained when the space distribution of flux wave along the air gap periphery is sinusoidal. But the air gap flux is not purely sinusoidal as it contains odd harmonics (5^{th}, 7^{th}, 11^{th} etc). Hence at low speeds, the torque-slip characteristic is not smooth. The distribution of stator winding and variation of air gap reluctance due to stator and rotor slots are main causes of air gap flux harmonics.

The harmonics caused due to variation of air gap reluctance are called tooth or slot harmonics . Due to these harmonics produced in air gap flux, unwanted torque are developed along with vibration and noise.

Now eventhough stator currents are sinusoidal, the stator m.m.f. is not sinusoidal as stator winding has the number of slots not more than 3 to 4 per phase. If carry out analysis of stator m.m.f. with the help of Fourier series it can be seen that in addition to fundamental wave it contains odd harmonics m.m.f. waves.

The third harmonic flux waves produced by each of the three phases neutralize each other as it differs in time phase by 120^o. Thus air gap flux does not contain third harmonics and its multiplies. The fundamental mmf wave produces flux which rotates at synchronous speed which given as $n_s = 2f_1/P$ rps where f_1 is supply frequency and P is number of poles. Similarly fifth harmonic mmf wave produces flux which rotates at $2f_1/5P = n_s/5$ rps and in direction opposite to the fundamental mmf wave. The seven harmonic mmf produces flux which rotates at $n_s/7$ rps and in the direction of fundamental m.m.f. wave.

Thus it can be seen that harmonic m.m.f. wave produces flux which rotates at 1/K times the fundamental speed and in the direction of fundamental wave if K = 6m + 1 and in the reversed direction if K = 6m - 1 where m is any integer. The most important and predominant harmonics whose effects must be studied are 5th and 7thharmonics.

The electromagnetic torque that is developed in the induction motor is because of zero relative speed between stator and rotor fields. This fact can be explained as follows :

When rotor is revolving in the same direction of rotation as the stator field, the frequency of rotor currents is sf_1 and the rotor field produced will have speed of sn_srpm with respect to rotor in the forward direction. But there is mechanical rotation of rotor at n rpm which is superimposed on this. The speed of rotor field in space is thus given by sum of these speeds

$$sn_s + n = sn_s + n_s(1-s) = n_s$$

The stator and rotor fields are thus stationary with respect to each other which produces a steady torque maintaining the rotation. This torque existing at any mechanical speed n other than synchronous speed is called synchronous torque.

The fifth harmonic field rotates at $n_s/5$ rps and in a direction opposite to direction of rotor. Therefore slip of rotor with respect to fifth harmonic field speed is

$$s_5 = \frac{n_{s\,\text{fifth harmonic}} - n_r}{n_{s\,\text{fifth harmonic}}} \quad \text{where } n_r \text{ is rotor speed.}$$

$$= \frac{-\frac{n_s}{5} - n_r}{-\frac{n_s}{5}} = \frac{-\frac{n_s}{5} - n_s(1-s)}{-\frac{n_s}{5}} = 1 + 5\,(1 - s) = 6 - 5\,s$$

Here $-n_s/5$ represents fifth harmonic field rotating opposite to the rotor. The frequency of rotor currents induced by fifth harmonic rotating field is

$$f_{2\,\text{fifth harmonic}} = s_5 \text{ x Stator frequency}$$

$$= (6 - 5s) \text{ x } f_1$$

Now speed of fifth harmonic rotor field with respect to rotor is given by

$$\frac{2\,(f_{2\,\text{fifth harmonic}})}{5\,p} = \frac{2}{5p}\,f_1\,(6-5s) = \frac{n_s}{5}\,(6-5s)$$

Now, speed of fifth harmonic rotor field with respect to stator

$$= \begin{array}{c}\text{Speed of fifth harmonic rotor}\\ \text{field with respect to rotor}\end{array} + \text{Rotor speed}$$

$$= -\frac{n_s}{5}\,(6-5s) + n_r = -\frac{6}{5}\,n_s + sn_s + n_s\,(1-s) = -\frac{n_s}{5}\ \text{rps}$$

Negative sign is used before $n_s/5\ (6 - 5s)$ which indicates 5th harmonic field rotates opposite to rotor movement. Thus it can be seen that speed of fifth harmonic stator field and rotor field is equal and relative speed between the two is zero. Thus it produces 5th harmonic induction motor torque similar to torque produced by fundamental component.

Similar analysis can be made on 7th harmonic to show 7th harmonic torque produced similar to fundamental one. Thus each space harmonic can be considered to produces its own asynchronous torque. The induction motor can be considered as equivalent to number of induction motors in series having poles equal to number of harmonics multiplied by number of poles. The torque produced by fundamental component and the harmonic are shown in the Fig. 2.

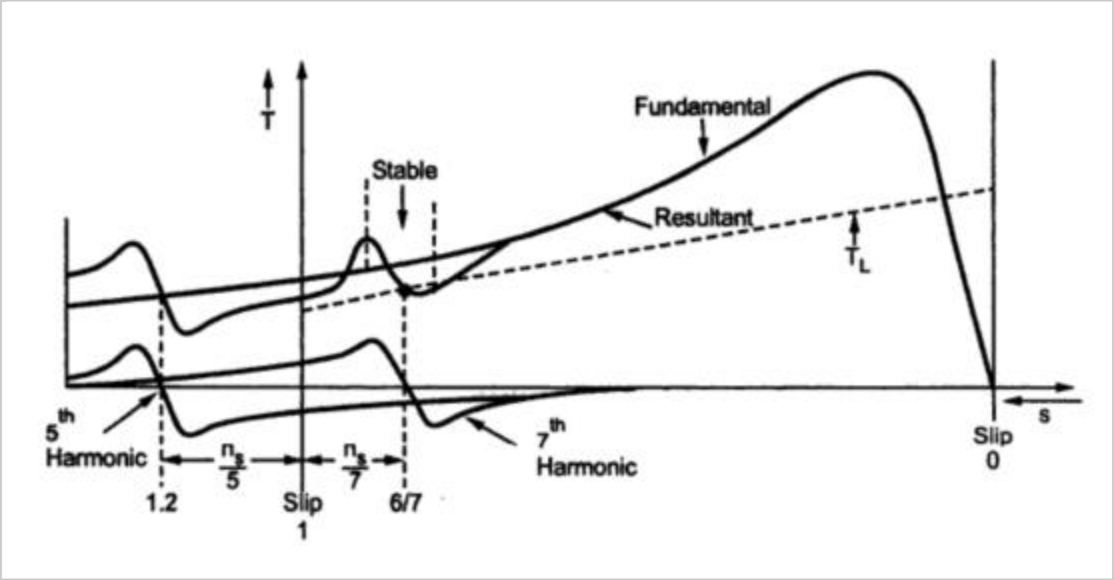

Fig. 2 Presence of harmonics

Crawling

As fifth harmonic field rotates opposite to the rotor rotation, the torque produced by fifth harmonic opposes fundamental torque and it acts as braking torque on motor. The seventh harmonic field rotates in the direction of rotor rotation, the torque produced by seventh harmonic aids the fundamental torque. The resultant torque is shown in the Fig. 2 which shows the addition of fundamental, fifth harmonic and seventh harmonic torque. The fifth harmonic torque is zero at $-n_s/5$ rps while seventh harmonic torque is zero at $+n_s/7$.

There are two dips which can be seen in the resultant torque, one is near the slip 1.2 and other near slip 6/7. The slip near s =6/7 is more important as torque here decreases with increase in speed. The load torque is shown in figure. The rotor will run at $n_s/7$ with X as the operating point. Thus stable operation is obtained near sub-synchronous speed $n_s/7$. This is called crawling or synchronous crawling. Due to crawling there is much higher stator current accompanied by noise and vibration. The torque obtained from induction motor here is called synchronous called.

When two harmonic fluxes of same order one because of stator and the rotor because of rotor interact with each other at one particular speed and produces harmonic synchronous torque just like that produced in synchronous motor. These torques are caused by tooth harmonics. The stable operation at synchronous speed caused by slot harmonics is called synchronous crawling which is associated with vibration and noise.

Cogging

A special behaviour is shown by squirrel cage induction motor during starting for certain combinations of number of stator and rotor slots. If number of stator slots $S_1$are equal to number of rotor slots S_2 or integral multiple of rotor slots S_2 then variation of reluctance as a function of space will have pronounced effect producing strong forces than the accelerating torque. Due to this motor fails to start. This phenomenon is called cogging. Such combination of stator and rotor slots should be avoided while designing the motor.

Let the slots of stator and rotor be 24. The stator-slotting produces its tooth harmonics of order $2S_1/P \pm 1$ whereas the rotor-slotting produces its tooth harmonics of order $2S_1/P \pm 1$ where S_1 and S_2 are number of stator and rotor slots. The plus sign refers to the harmonic field rotation in the direction of rotor.

Here $S_1 = S_2$ so stator and rotor slot harmonics are same and given by,

Let $P = 4$

$$(2x24/ 4) \pm 1 = 11 \text{ or } 23$$

The harmonics of order 11 produce backward rotating field for both stator and rotor. The harmonics of order 13 produces forward rotating field.

The two harmonics fields of same order say 11th harmonic would be stationary with respect to each other only when

$$n_r - (n_s-n_r / 11) = -n_s/11$$

$$n_r = 0$$

As the harmonic field due to 11th harmonic rotates backward with respect to stator hence negative sign is used for $n_s/11$.

Similarly, for 13th harmonic produced by stator and rotor would be stationary with respect to each other when

$$= (n_s-n_r / 13) + n_r = n_s/13$$

$$n_r = 0$$

Hence it can be seen that harmonic synchronous torque is produced at zero rotor speed. The 11th and 13th harmonic fields produced by stator and rotor and stationary with respect to each other. The harmonic synchronous torque is produced at zero rotor speed and the motor will remain at rest. This is called cogging. The torque speed characteristic with harmonic synchronous torque as $n_s/7$ is shown in the Fig.3.

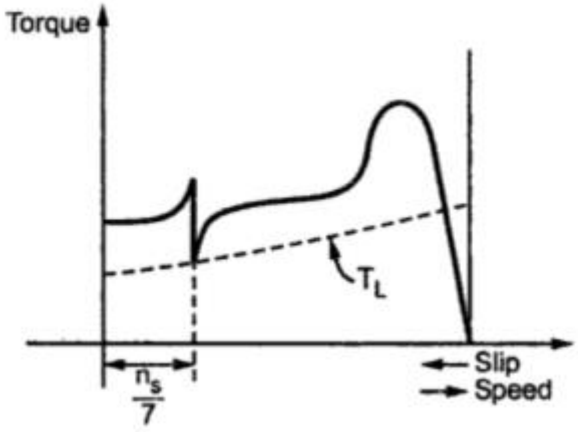

Fig. 2 Cogging

The stator slot harmonics of order $2S_1/P \pm 1$ may interact with rotor slot harmonics of order $2S_2/P \pm 1$ to develope the harmonic synchronous torques.

$$2S_1/P + 1 = 2S_2/P + 1$$

$$S_1 = S_2$$

And $$2S_1/P - 1 = 2S_2/P + 1$$

$$S_1 - S_1 = P$$

It can be thus seen that if $S_1 = S_2$ or $S_1 - S_2 = P$ then cogging will be definately observed in the induction motor.

The cogging and crawling is not predominately in slip ring induction motor as these motors are started with higher starting torques with external resistance in rotor circuit.

The crawling effect can be reduced by taking proper car during the design. Still if crawling is observed then it can be overcome by applying a sudden external torque to the driven load in the direction of rotor. If there is reduction in supply voltage then torque also decreases ($T \alpha V_1^2$). Hence asynchronous crawling may be observed which is absent under rated voltage conditions. Thus asynchronous torques can not be avoided but can be reduced by proper choice of coil span and by skewing the stator or rotor slots.

Key Point : The synchronous harmonics torques can be totally eliminated by proper combination of stator and rotor slots.

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Key Point : The synchronous harmonics torques can be totally eliminated by proper combination of stator and rotor slots.

Special Rotor Constructions and Applications

In case of slip ring induction motor an external resistance can be added in the rotor circuit during starting which gives higher starting torque and lower starting line current at an improved power factor. This resistance is then gradually cut from the rotor circuit which would otherwise result in decrease of full load speed, poor speed regulation, more rotor losses and hence reduced efficiency. With lower rotor resistance it gives constant speed, low slip, less losses and high efficiency. This is the major advantage of slip ring induction motor that it gives high rotor resistance at starting and low rotor resistance at normal operating speed.

In case of squirrel cage induction motor there is no provision made for adding external resistance. If the resistance is designed in such a way that it gives better running performance then it has high starting current and consequently low starting torque. This is major disadvantage of squirrel cage induction motor although it is having the other qualities of low cost, ruggedness and maintenance free operation. Thus the designer had found different ways of improving the starting performance of the motor without affecting the running performance of the motor.

In squirrel cage induction motor high starting torque can be obtained by the use of deep bar or double cafe rotors. Both these types of rotors make use of skin effect in which distribution of current is not uniform but the alternating current has the tendency to concentrate near the surface of the conductor. Due to this effect, effective area of cross section of the conductor is reduced and hence resistance of the conductor is increased when carrying alternating current.

The solid conductor can be considered to be make up of large number of strands each carrying a small part of current. The inductance of each strand will vary according to the position. The strands in proximity of the centre are surrounded greater magnetic flux and has greater inductance than near the surface. Due to high reactance at the centre, the alternating current flows near the surface of the conductor. The skin effects depends upon nature of material, diameter of wire, shape of wire and frequency.

Thus the current in the rotor during starting is having the frequency of supply. While under running condition the frequency of rotor current reduces to slip frequency. This variation in frequency changes the rotor resistance as it depends on skin effect. During starting it gives high resistance whereas it gives low resistance during running condition which is desirable. Thus the variation in rotor resistance can be achieved by deep bar or double cage construction of rotor and induction motor. Both these types of construction make use of skin effect phenomenon.

Deep Bar Rotor Construction

There is no constructional difference between stator of deep bar motor and that of ordinary induction motor. The rotor consists of deep bars, short circuited by two end rings one on each side. The deep and narrow rotor bar of rectangular cross section is shown in the Fig. 1(a). The other rotor bar shapes are shown in the Fig. 1(b). The magnetic leakage flux lines are shown by dotted lines, Now consider that the bar consists of many number of layers of different depths. The top and bottom layers are shown in the Fig. 1.

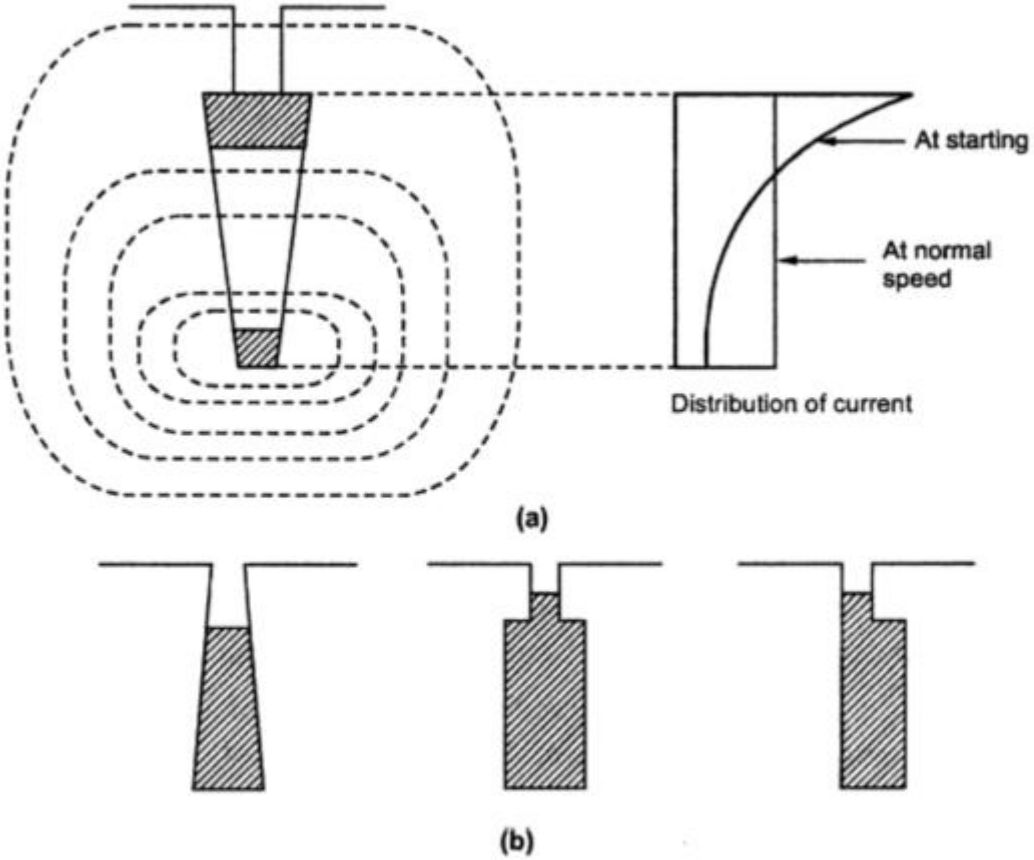


Fig. 1 Deep bar rotor

The leakage inductance of the bottom strips is greater than that of top strips as more flux links with bottom strip compared to top strip. All the strips are in parallel electrically. The bottom strip has greater leakage inductance than the top strip. During starting the rotor frequency is equal to the stator frequency and hence leakage reactance of bottom strip is largest and current in it is least. The top strip has low leakage reactance and current in it is large. Thus the current in low reactance top strip will be greater than that in high reactance lower strip and the current will be forced towards the top of the slot and phase of current in upper strip will lead that of the current in lower one. Thus there is non-uniform distribution of current which is shown in the Fig. 1. Due to this non-uniform distribution of current, and use to skin effect, effective are of cross section decreases. Hence rotor resistance increases resulting i high starting torque.

As leakage reactance is proportional to frequency, the non-uniform distribution of current depends upon the rotor frequency. The Fig. 2 shows a curve indicating a.c. effective resistance to d.c. resistance with change in frequency for a copper bar of 2.5 cm deep. The skin effect is maximum when rotor is at standstill.

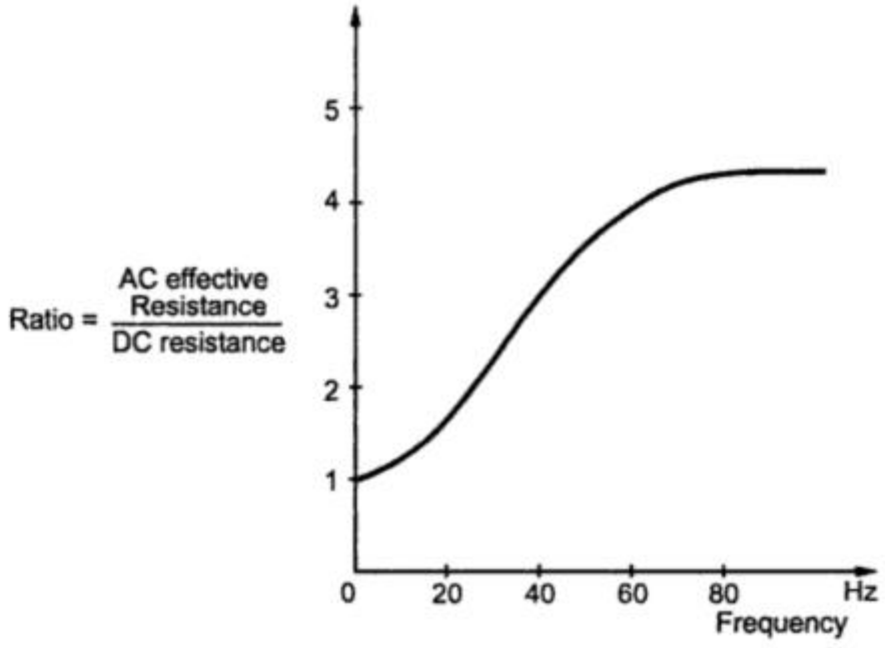


Fig. 2

With the increase in rotor speed, the rotor frequency decreases and skin effect also decreases. The reactances of different strips at this low frequency become almost equal and the current density over the conductor cross section becomes uniform so its a.c. resistance is equal to d.c. resistance. Thus with deep bar rotor has a low starting current with high starting torque without affecting running performance of motor. The net reactance of deep bar rotor at standstill is higher than that in a normal bar design, the breakdown or pull out torque in deep bar rotor is lower. The torque-slip characteristics of deep bar motor and normal induction motor is shown in the Fig.3.

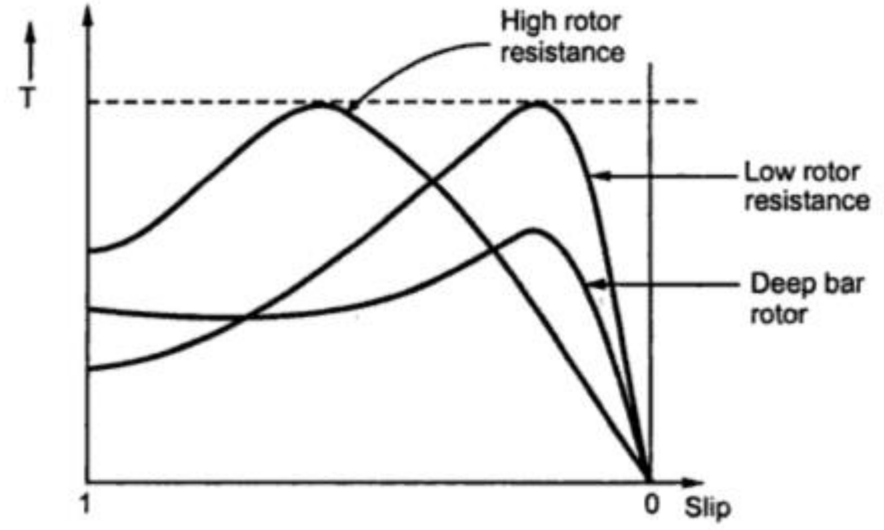


Fig. 3 Torque slip characteristics of deep bar rotor

The equivalent circuit of induction motor is applicable to deep bar rotor also wherein proper value of r_2' and x_2' must be determined for satisfactory running performance. During starting their values should correspond to effective value at stator frequency. During running their values should correspond to their effective values at low rotor frequency.

Double Cage Rotor Construction

This is another way of obtaining improved starting performance without affecting its running performance. Though it is more expensive it gives better performance than deep bar rotor construction.

The stator of double cage rotor induction motor is same as that of ordinary induction motor whereas its rotor consists of two cages or two layers of bars short circuited by end rings since the upper cage is having smaller cross-sectional area than the lower cage, the upper cage is having higher resistance than that of lower cage. With equal cross sectional areas of two cages the upper cage is made up of high resistance material like brass, aluminium, bronze etc. and the lower cage is made up of low resistance material like copper. The upper cage and lower cage are separated by a narrow slit or constriction. This is shown in the Fig. 4.

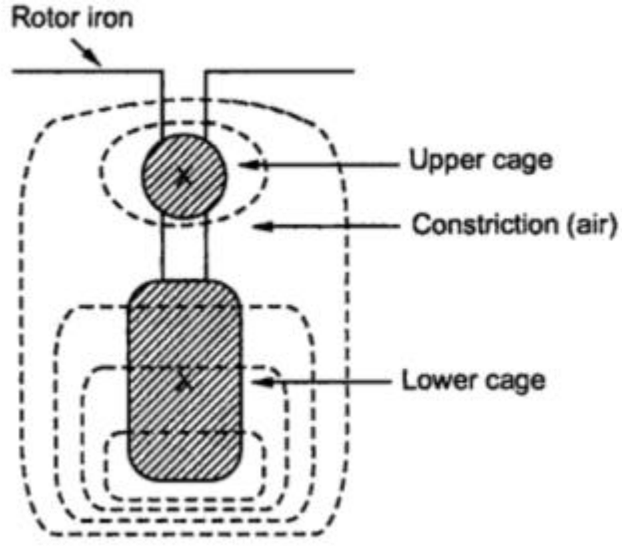


Fig. 4 Double cage rotor construction

The slot leakage flux pattern is also shown in the Fig. 4 for the double cage rotor. Similar to deep bar rotor construction the rotor bars in the upper cage have less leakage flux linkage and therefor has lower reactance. The dimension of air construction controls the self leakage flux linking upper and lower bars. If air constriction would have been absent then the main flux would return via iron path between the two slots and thereby missing the bars in the lower cages which will not contribute to production of torque in that case. Hence it can be seen that the upper cage has high resistance and low reactance whereas the lower cage has low resistance and high reactance.

During starting the rotor frequency is same as stator frequency or supply frequency. The division of rotor current in upper and lower cage is inversely proportional to their leakage impedances. At the time of starting the leakage reactance of lower cage is very high and consequently its leakage impedance is several times greater than that of upper cage whose leakage reactance is small. Hence most rotor current flows in upper cage having lower leakage impedance. The upper cage having high resistance sharing the rotor current results in low starting current at improved power factor giving high starting torque.

When rotor speeds up, the rotor frequency decreases which decreases the leakage reactance of lower cage. At normal operating speed the reactance difference between the two cages is negligibly small. Hence the division of rotor current in this case is mainly decided by the resistances of the two cages. As resistance of upper cage is very high most of the current flows through the lower cage giving excellent operating characteristics under running condition. It can be noted that starting current is confined mainly with upper cage so if there is frequent starting of motor then it would cause overheating and buring of upper cage.

The torque-slip characteristics of double cage induction motor are shown in the Fig. 5.

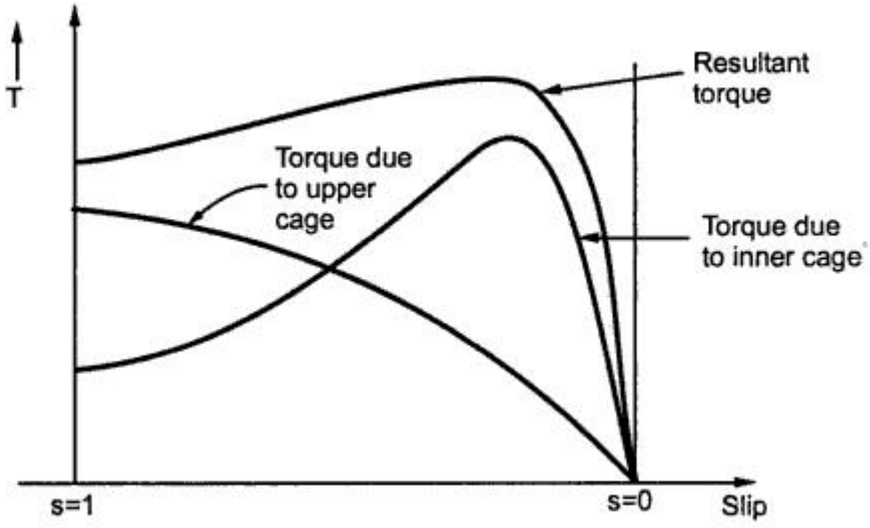


Fig. 5 Torque slip characteristics of double cage induction motor

Another type of double cage rotor construction is also possible which is shown in the Fig. 6. The slot-leakage flux pattern for this type of construction is also shown.

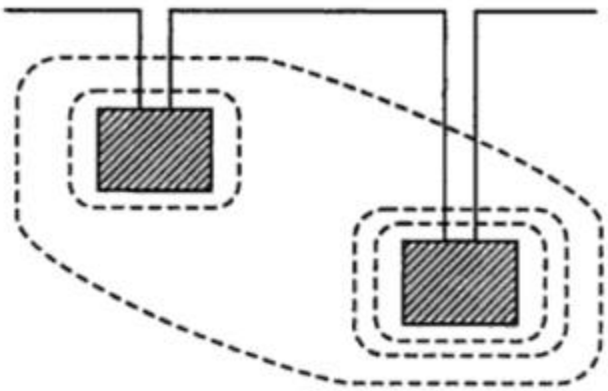


Fig. 6

The approximate equivalent circuit of double cage rotor induction motor is shown in the Fig. 7. Though the two cages are somewhat coupled magnetically, they can be treated as independent for simplicity and it gives approximately same results. The two cages are assumed to be parallel while drawing the equivalent circuit.

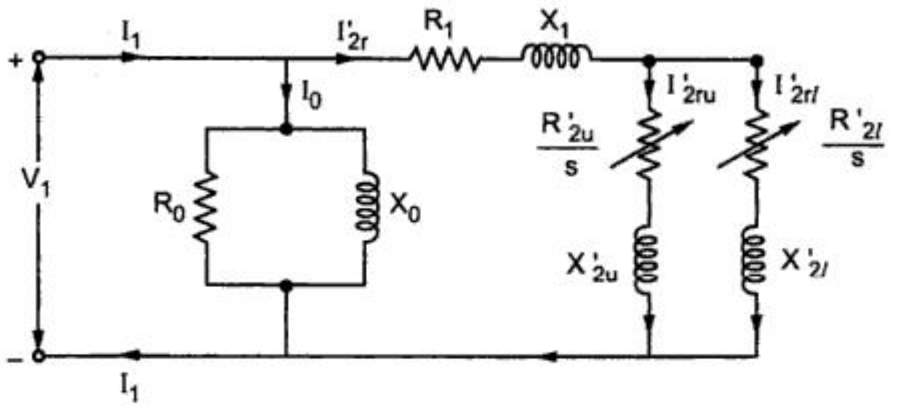
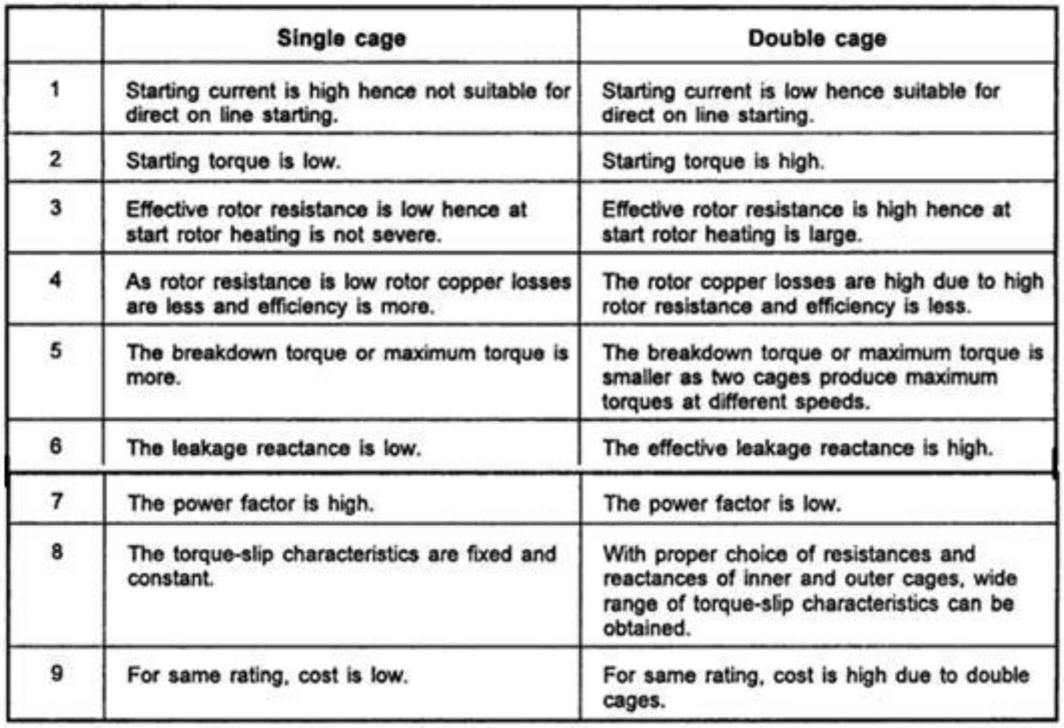


Fig. 8 Equivalent circuit of double cage induction motor

I_{2ru}' and I_{2r}' are the currents in the upper and lower cages respectively referred to the stator $R_{2u}'$and R_{2e}' are the resistance of upper and lower cages referred to the stator whereas and are leakage reactances of the two cages referred to the stator of the motor.



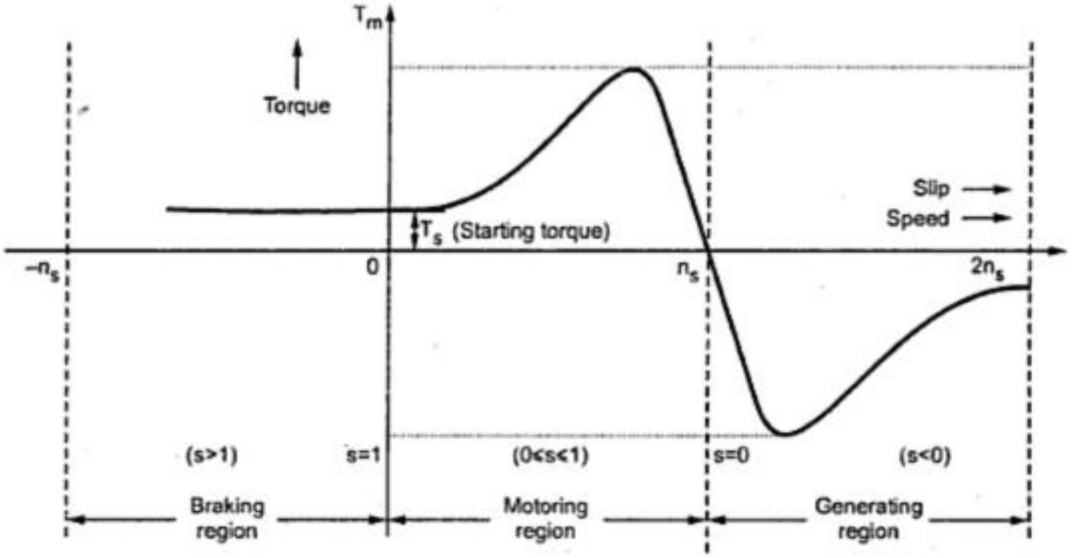
	Single cage	Double cage
1	Starting current is high hence not suitable for direct on line starting.	Starting current is low hence suitable for direct on line starting.
2	Starting torque is low.	Starting torque is high.
3	Effective rotor resistance is low hence at start rotor heating is not severe.	Effective rotor resistance is high hence at start rotor heating is large.
4	As rotor resistance is low rotor copper losses are less and efficiency is more.	The rotor copper losses are high due to high rotor resistance and efficiency is less.
5	The breakdown torque or maximum torque is more.	The breakdown torque or maximum torque is smaller as two cages produce maximum torques at different speeds.
6	The leakage reactance is low.	The effective leakage reactance is high.
7	The power factor is high.	The power factor is low.
8	The torque-slip characteristics are fixed and constant.	With proper choice of resistances and reactances of inner and outer cages, wide range of torque-slip characteristics can be obtained.
9	For same rating, cost is low.	For same rating, cost is high due to double cages.

2. Application

i) Squirrel cage type of motors having moderate starting torque and constant speed characteristics preferred for driving fans, blowers, water pumps, grinders, lathe machines, printing machines, drilling machines.
ii) Slip ring induction motors can have high starting torque as high as maximum torque. Hence they are preferred for lifts, hoists, elevators, cranes, compressor.

Induction Generator

The torque-slip or the torque-speed characteristics of the induction motor are shown in the Fig. 1. The operating mode of induction machine as a generator or motor or braking depends on value of slip s.



Regions of torque - slip characteristics

Generating and Braking Region

When the slip lies in the region 0 and 1 i.e. when $0 \leq s \leq 1$, the machine runs as a motor which is the normal operation. The rotation of rotor is in the direction of rotating field which is developed by stator currents. In this region it takes electrical power from supply lines and supplies mechanical power output. The rotor speed and corresponding torque are in same direction.

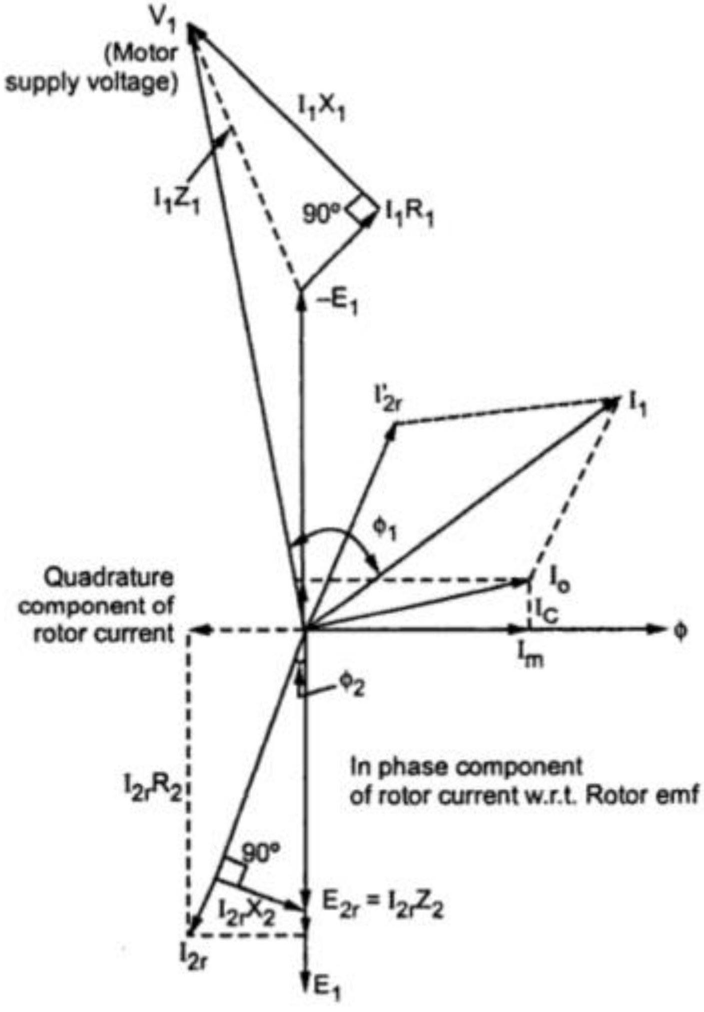
When the slip is greater than 1, the machine works in the braking mode. The motor is rotated in opposite direction to that of rotating field. In practice two of the stator terminals are interchanged which changes the phase sequence which in turn reverses the direction of rotation of magnetic field. The motor comes to quick stop under the influence of counter torque which produces braking action. This method by which the motor comes to rest is known as plugging. Only care is taken that the stator must be disconnected from the supply to avoid the rotor to rotate in other direction

To run the induction machine as a generator, its slip must be less than zero i.e. negative. The negative slip indicates that the rotor is running at a speed above the synchronous speed. When running as a generator it takes mechanical energy and supplies electrical energy from the stator.

Thus the negative slip, generation action takes place and nature of torque - slip characteristics reverses in this generating region.

The construction of induction generator is same as that of motor with the difference that the direction of rotation of the motor and a generator is opposite for the same current direction.

The action of induction machine as a generator can be explained from the phasor diagram.



The induction generator is not self excited as it can not generates its own exciting current. Thus it must be always connected to an a.c. supply. Generally it is operated in parallel with synchronous machines. It is shown in the Fig.3.

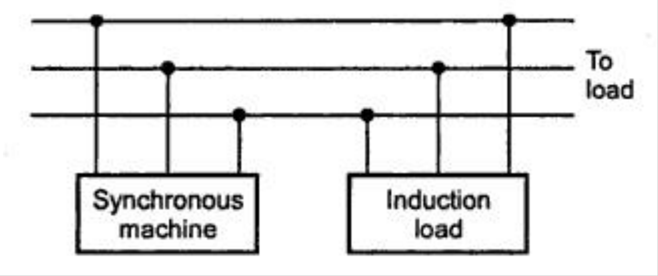


Fig. 3

Consider an example of a load which requires a lagging current which can not be supplied by induction generator alone as it supplies leading current.

But this current requirement is fulfilled with the help of synchronous generators operating in parallel with induction generator. Consider the following phasor diagram.

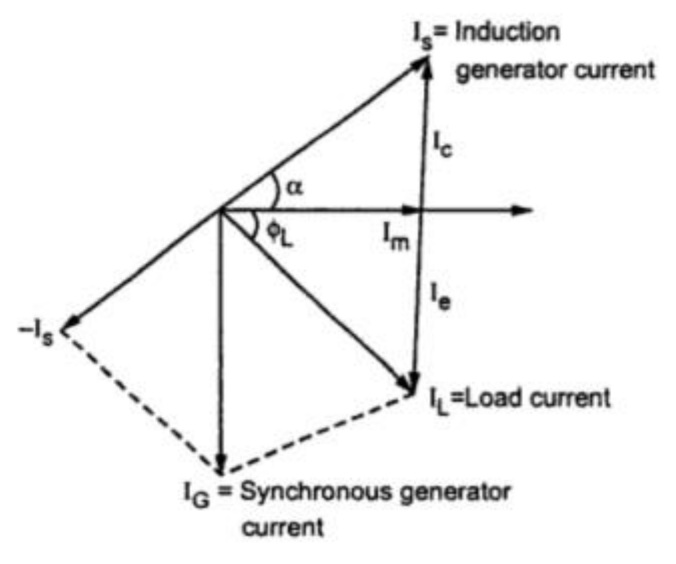


Fig. 4

The load current I_L can be resolved into two components one in phase component I_m and the other quadrature component I_e. The speed of the induction generator is adjusted in such a way that it supplies current I_c which is leading one. The induction generator current I_s is nothing but vector sum of I_c and I_m.

The synchronous generator which is in parallel with the induction generator must supply the remaining part of load current. For this the induction generator current I_s is subtracted vectorially from I_L (subtracting vectorially means reversing I_s and adding it with I_L). This current is nothing but algebraic sum of currents I_c and I_e. The synchronous generator supplies no power. The total current supplied by synchronous generator is lagging quadrature current.

If the load requires a leading current then theoretically the quadrature component of current can be supplied entirely by the induction generator. But for satisfactory operation it should be run in parallel with synchronous generator.

If the bank of delta connected capacitors is operated in parallel with induction generator then the reactive power requirement of induction generator is met by capacitors. This arrangements is shown in Fig. 5.

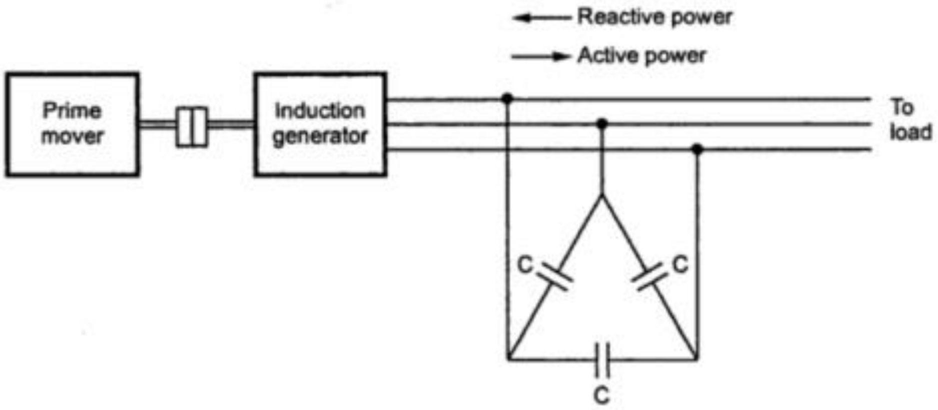


Fig. 5

The induction generator in this case is said to be isolated induction generator supplying a load. The external voltage source is not required in this case.

Unlike in synchronous generators, induction generators are not rotating at a definite speed at a given frequency. The speed varies with load as the load is proportional to slip. The frequency of the induction generator is same as the frequency of the line to which it is connected.

Comparison of Induction Generator and Synchronous Generator

The distinct features of induction generator compared to synchronous generators are as follows :

i) It will not require d.c. excitation.

ii) It is not self excited but external a.c. supply of fixed frequency is required.

iii) The frequency of induction generator is decided by the frequency of the excitation voltage which is supplying current to it.

iv) Synchronization of generator is not required as no emf is generated until it is connected to the line.

Advantages

The following are the advantages of induction generator.

i) Synchronization for induction generator is required.

ii) The construction is rugged for rotating parts.

iii) Unlike in synchronous machine, there is no danger of hunting or drop out of synchronism for induction generators.

iv) When it short circuited, it delivers small power as the excitation quickly reduces to zero.

v) Induction generators are more suitable for high speeds.

vi) With the help of excitation supply and frequency, the voltage and frequency of induction generator are controlled.

Disadvantages

Although induction generators are having above mentioned advantages, it has following advanatges.

i) It must be run in parallel with the synchronous machine.

ii) The load is not deciding the power factor of induction generator but the power factor depends on slip.

Applications

Because of distinct superiority of the synchronous generator, induction generators are rarely used to supply commercial power.

One application of induction generator is in railway for braking purposes. When the train is moving down a gradient, the induction generators runs above synchronism. As the torque in this region is negative, the braking action is achieved in the train. In addition to this the energy generated by induction generator is given to the line so that the load on main generating station is somewhat relieved. In this case no complicated control apparatus is required.

Circle Diagram : Introduction

In a particular circuit, if one of the circuit elements is variable, then depending upon its value, the circuit characteristics varies. As the value of the variable element is changed, the circuit parameters like current, power factor, power losses etc. also change. The locus of the extremity of the current phasor, obtained for various values of a variable element is called a locus diagram.

From the equivalent circuit of an induction motor, the motor can be treated as series R-L circuit where the element resistance of the circuit is variable which varies as slip s. Thus for variable load conditions, the resistance changes and hence the current drawn by the motor also changes. The locus diagram of such a current phasor is circular in nature and hence called circle diagram of a three phase induction motor. Using this diagram, all the performance characteristics of an induction motor like power factor, efficiency, stator losses, rotor losses, maximum output, maximum torque etc. can be predicted. Thus, a circle diagram is a graphical approach of predetermining the operation characteristics of an induction motor

Circle Diagram for a Series R-L Circuit

Consider a series R-L circuit with a variable R as shown in the Fig. 1. It is excited by an alternating source of V volts. The frequency of the source is f Hz.

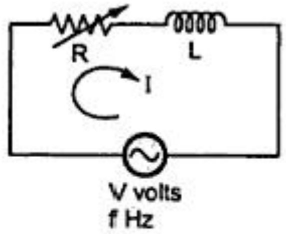


Fig. 1

Let I = Current flowing through the circuit

Z = Impedance of the circuit

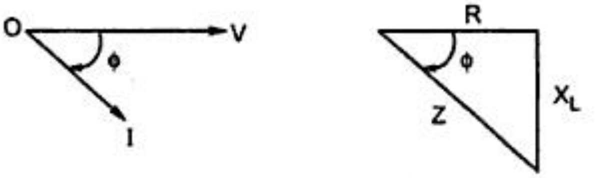
$Z = R + j X_L$ where $X_L = 2 fL$

Now R is variable while X_L is fixed.

$$\therefore \quad I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$= \frac{V}{X_L} \cdot \frac{X_L}{\sqrt{R^2 + X_L^2}} \quad \text{... Multiply and divide by } X_L$$

The phasor diagram is shown in the Fig. 2(a). The current I lags voltage V by angle as the circuit is inductive. The impedance triangle is shown in the Fig. 2(b).



(a) Phasor diagram **(b) Impedance triangle**

From the impedance triangle we can write,

$$\sin \Phi = X_L/Z$$

Substituting in the expression for I,

$$I = (V/X_L) \sin\Phi \qquad \text{.............(1)}$$

This is the equation of a circle in polar co-ordinates with a diameter equal to (V/X_L).

When the resistance R =0, then $\Phi = 90^o$ hence $\sin \Phi = 1$.

$$\therefore \qquad I = I_m = (V/X_L)$$

This is the maximum value of current.

As R resistance, the phase angle decreases thus decreasing sin. Effectively current I also decrease. When R→ ∞ the Φ→0° and current becomes zero.

The locus obtained of extremities of a current phasor plotted for various values of R is a semicircle. The semicircle is shown in the Fig. 3. The voltage axis is taken as vertical axis as a reference, with respect to which the various current phasors are plotted.

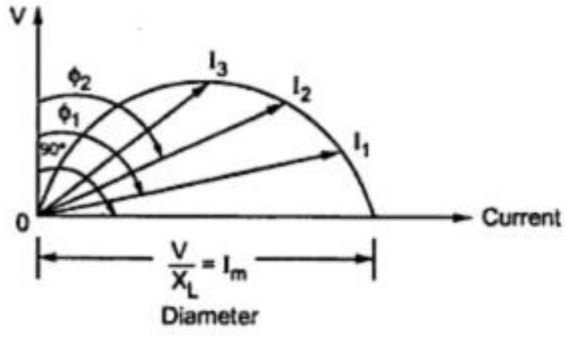


Fig. 3 Circle diagram

The power factors at various conditions are $\cos\Phi_1$, $\cos\Phi_2$ etc. As Φ varies only from $0^o$to 90^o, the diagram is semicircle, infact it is a half part of a circle hence it is known as circle diagram.

This theory of series R-L circuit can be easily extended to a three phase induction motor.

Circle Diagram of a 3 Phase Induction Motor

The equivalent circuit of a 3 phase induction motor is shown in the Fig.1.

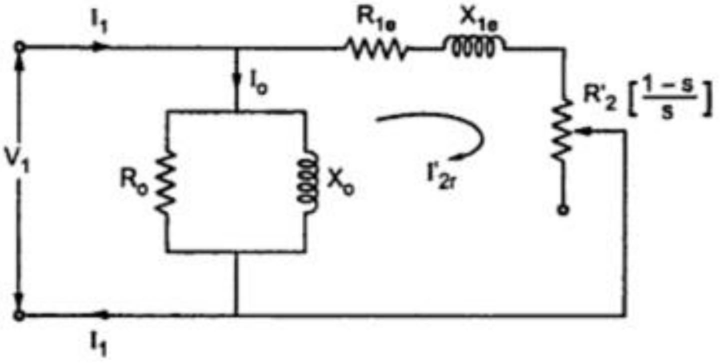


Fig. 1 Equivalent circuit of a 3 phase induction motor

All the values shown are per phase values. The circuit is similar to series R-L circuit. The reactance X_{1e} is fixed while the total resistance $R_{1e} + (R_2'(1-s)/s)$ is variable. This is because the slip s varies as load varies. The voltage across the parallel exciting branch is V_1. Hence we can write the expression for the rotor current referred to stator as,

$$I_{2r}' = V_1/\sqrt{((R_{1e} + R_L)^2 + X_{1e}^2)}$$

Where $R_L' = R_2' (1-s)/s$ = Variable equivalent load resistance

$R_{1e} = R_1 + R_2'$ = Equivalent resistance of motor referred to stator

$X_{1e} = X_1 + X_2'$ = Equivalent reactance of motor referred to stator

Dividing and multiplying by,

$$I'_{2r} = \frac{V_1}{X_{1e}} \times \frac{X_{1e}}{\sqrt{(R_{1e} + R'_L)^2 + (X_{1e})^2}}$$

$$\therefore \quad I_{2r}' = I_{max} \sin \qquad \text{....................(1)}$$

where $\sin\Phi = X/Z = X_{1e}/\sqrt{((R_{1e} + R_L')^2 + X_{1e}^2)}$

and $I_{max} = V_1/X_{1e}$

The I_{2r}' will be at its maximum when $R_{1e} + R_L' = 0$ i.e., there exists an ideal short circuit. Hence current I_{max} is called ideal short circuit current of an induction motor.

The equation (1) represents equation of a circle with as its diameter. Thus locus of extremely of I_{2r}' is a circle, as shown in the Fig.2.

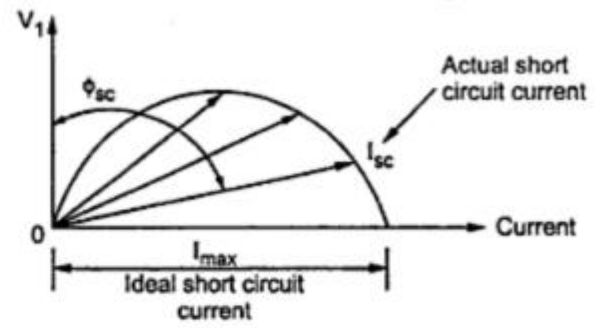


Fig. 2 Circle diagram of I_{2r}'

But the total stator current I_1 per phase is a vector addition of current I_o and I_{2r}'.

$I_1 = I_o + I_{2r}'$ .... Vector addition

For and induction motor, I_o has a fixed value and phase angle Φ_o which is decided by its active component and magnetising component I_m.

$$\bar{I}_o = \bar{I}_c + \bar{I}_m$$

As I_o has fixed magnitude and phase, the locus of extremities of I_1, which is $I_o + I_{2r}'$ is also a circle with a diameter still as V_1/X_{1e}. The only charge will be that the diameter V_1/X_{1e} will no longer be along X-axis i.e. current axis but will get shifted at the tip of the I_o phasor. All the I_{2r}' phasors are to be drawn from I_o phasor to get I_1, as has fixed magnitude and phase angle Φ_o.

Key Point : Thus the current locus for a stator current is also a semicircle which is truly called circle diagram of a three phase induction motor. This diagram once obtained can be used to predict the performance of an induction motor under variable load conditions.

The circle diagram is shown in the Fig. 3.

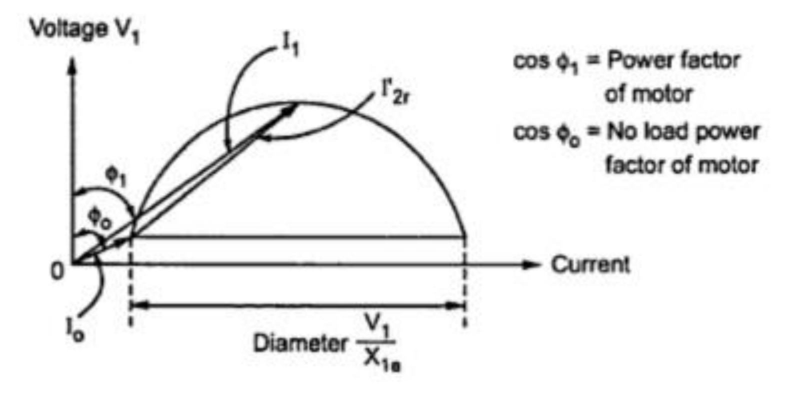


Fig. 3 Circle diagram of a three phase induction motor

Let us see, how to obtain the data for plotting the circle diagram.

Obtaining Data to Plot Circle Diagram

The data required to draw the circle diagram is obtained by conducting two testes which are,

1. No load test or open circuit test

2. Blocked rotor test or short circuit test

No Load Test

In this test, the motor is made to run without any load i.e. no load condition. The speed of the motor is very close to the synchronous speed but less than the synchronous speed. The rated voltage is applied to the stator. The input line current and total in put power is measured. The two wattmeter method is used to measure the total input power. The circuit diagram for the test is shown in the Fig. 1.

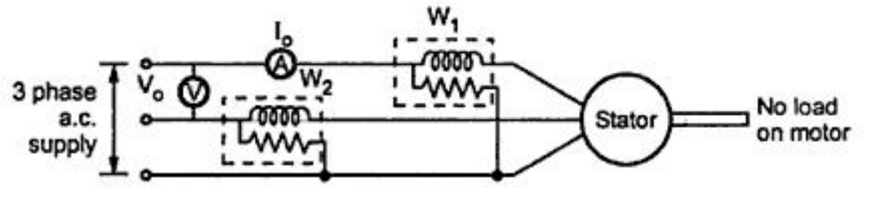
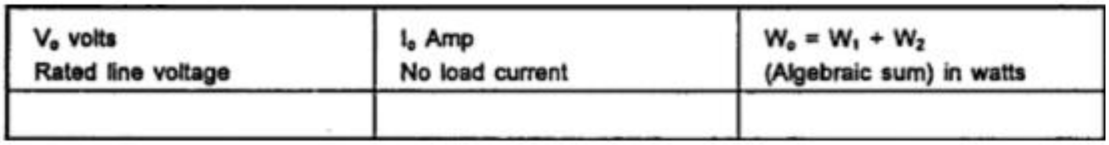


Fig. 1 No load test

As the motor is on no load, the power factor is very low which is less than 0.5 and one of the two wattmeters reads negative. It is necessary to reverse the current coil or pressure coil connections of such a wattmeter to get the positive reading. This reading must be taken negative for the further calculations.

The total power input W_o is the algebraic sum of the two wattmeter readings. The observation table is,



V_o volts Rated line voltage	I_o Amp No load current	$W_o = W_1 + W_2$ (Algebraic sum) in watts

The calculations are,

$$W_o = \sqrt{3} V_o I_o \cos\Phi_o$$

$\therefore$ $\cos\phi_o = \dfrac{W_o}{\sqrt{3}\, V_o I_o}$ where V_o, I_o are line values

This is no load power factor.

Thus we are now in a position to obtain magnitude and phase angle of no load current I_o, which is required for the circle diagram.

From the knowledge of I_o and Φ_o, the parameters of the equivalent circuit can be obtained as,

$I_c = I_o \cos\Phi_o$ = Active component of no load current

$I_m = I_o \sin\Phi_o$ = Magnetising component of no load current

$R_o = V_o$ (per phase) / I_c (per phase) = No load branch resistance

$X_o = V_o$ (per phase)/ I_m (per phase) = No load branch resistance

The power input W_o consists of following losses,

1. Stator copper loss i.e. $3 I_o^2 R_1^2$where I_o is no load per phase current and R_1 is stator resistance per phase.

2. Stator core loss i.e. iron loss.

3. Friction and windage loss.

The no load rotor current is very small and hence rotor copper loss is negligibly small. The rotor frequency is s times supply frequency and on no load it is very small. Rotor iron losses are proportional to this frequency and hence are negligibly small.

Key Point : Under no load condition, I_o is also very small and in many practical cases it is also neglected.

Thus W_o consists of stator iron loss and friction and windage loss which are consists for all load condictions. Hence W_o is said to give fixed losses of the motor.

$\therefore$ W_o = No load power input

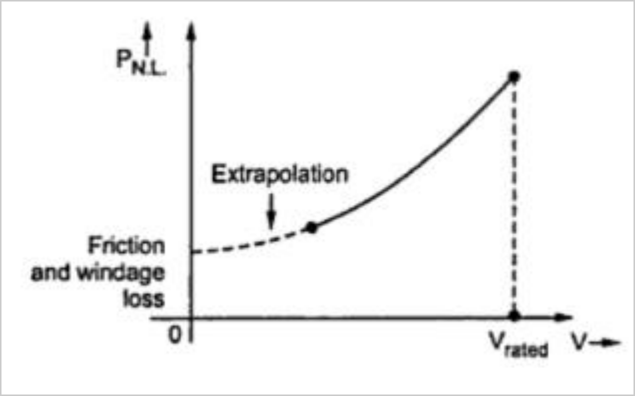
$\therefore$ $\boxed{W_o = \text{Fixed Loss}}$ ... Neglecting stator copper loss

Separation No Load Losses

The no load losses are the constant losses which include core loss and friction and windage loss. The separation between the two can be carried out by the no load test conducted from variable voltage, rated frequency supply.

When the voltage is decreased below the rated value, The core loss reduces as nearly square of voltage. The slip does not increase significantly the friction and windage loss almost remains constant.

The voltage is continuously decreased till the machine slip suddenly begins to increase and the motor tends to stall. At no load, this takes place at a sufficiently reduced voltage. The graph showing no load losses $P_{N.L.}$ versus as shown in the Fig. is extrapolated to V = 0 which gives friction and windage loss as iron or core loss is zero at zero voltage.



Blocked Rotor Tests

In this test, the rotor is locked and it is not allowed to rotate. Thus the slip s = 1 and $R_L' = R_2'(1-s)/s$ is zero. If the motor is slip ring induction motor then the windings are short circuited at the slip rings.

The situation is exactly similar to the short circuit test on transformer. If under short circuit condition, if primary is excited with rated voltage, a large shot circuit current can flow which is dangerous from the windings point of view. So similar to the transformer short circuit test, the reduced voltage (about 10 to 15 % of rated voltage) just enough such that stator carries rated current is applied. Now the applied voltage V_{sc}, the input power W_{sc} and a short circuit current I_{sc} are measured.

As $R_L' = 0$, the equivalent circuit is exactly similar to that of a transformer and hence the calculations are similar to that of short circuit test on a transformer.

V_{sc} = Short circuit reduced voltage (line value)

I_{sc} = Short circuit current (line value)

W_{sc} = Short circuit input power

Now $W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos\Phi_{sc}$ ....Line values

$$\therefore \quad \boxed{\cos\phi_{sc} = \frac{W_{sc}}{\sqrt{3}\, V_{sc}\, I_{sc}}}$$

This gives us short circuit power factor of a motor.

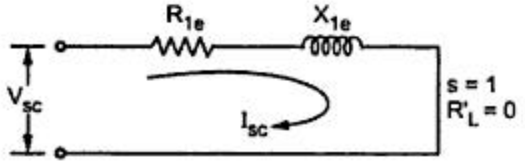


Fig. 1

Now the equivalent circuit is as shown in the Fig. 1.

$$\therefore \quad \boxed{W_{sc} = 3\,(I_{sc})^2\, R_{1e}}$$

where I_{sc} = Per phase value

∴	$R_{1e} = \frac{W_{sc}}{3\,(I_{sc})^2}$

This is equivalent resistance referred to stator.

$Z_{1e} = V_{sc}$ (per phase)/ I_{sc} (per phase) = Equivalent impedance referred to stator.

∴	$X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$ = Equivalent reactance referred to stator

During this test, the stator carries rated current hence the stator copper loss is also dominant. Similarly the rotor also carries short circuit current to produce dominant rotor copper loss. As the voltage is reduced, the iron loss which is proportional to voltage is negligibly small. The motor is at standstill hence mechanical loss i.e. friction and windage loss is absent. Hence we can write,

W_{sc} = Stator copper loss + Rotor copper loss

But it is necessary to obtain short circuit current when normal voltage is applied to the motor. This is practically not possible. But the reduced voltage test results can be used to find current I_{SN} which is short circuit current if normal voltage is applied.

If V_L = Normal rated voltage (line value)

V_{sc} = Reduced short circuit voltage (line voltage)

then	$I_{SN} = \left(\frac{V_L}{V_{sc}}\right) \times I_{sc}$

where I_{sc} = Short circuit current at reduced voltage

Thus, I_{SN} = Short circuit current at normal voltage

Now power input is proportional to square of the current.

So W_{SN} = Short circuit input power at normal voltage

This cab be obtained as,

$$\boxed{W_{SN} = \left(\frac{I_{SN}}{I_{sc}}\right)^2 W_{sc}}$$

But at normal voltage core loss can not be negligible hence,

W_{SN} = Core loss + Stator and rotor copper loss

Construction of Circle Diagram

By using the data obtained from the no load test and the blocked rotor test, the circle diagram can be drawn using the following steps :

Step 1 : Take reference phasor V as vertical (Y-axis).

Step 2 : Select suitable current scale such that diameter of circle is about 20 to 30 cm.

Step3 : From no load test, I_o and are Φ_o obtained. Draw vector I_o, lagging V by angle Φ_o. This is the line OO' as shown in the Fig. 1.

Step 4 : Draw horizontal line through extremity of I_o i.e. O', parallel to horizontal axis.

Step 5 : Draw the current I_{SN} calculated from I_{sc} with the same scale, lagging V by angle Φ_{sc}, from the origin O. This is phasor OA as shown in the Fig. 1.

Step 6 : Join O'A is called output line.

Step 7 : Draw a perpendicular bisector of O'A. Extend it to meet line O'B at point C. This is the centre of the circle.

Step 8 : Draw the circle, with C as a center and radius equal to O'C. This meets the horizontal line drawn from O' at B as shown in the Fig. 1.

Step 9 : Draw the perpendicular from point A on the horizontal axis, to meet O'B line at F and meet horizontal axis at D.

Step 10 : Torque line.

The torque line separates stator and rotor copper losses.

Note that as voltage axis is vertical, all the vertical distances are proportional to active components of currents or power inputs, if measured at appropriate scale.

Thus the vertical distance AD represents power input at short circuit i.e. W_{SN}, now which consists of core loss and stator, rotor copper losses.

Now FD = O'G

= Fixed loss

Where O'G is drawn perpendicular from O' on horizontal axis. This represents power input on no load i.e. fixed loss.

Hence AF α Sum of stator and rotor copper losses

Then point E can be located as,

AE/EF = Rotor copper loss / Stator copper loss

The line O'E under this condition is called torque line.

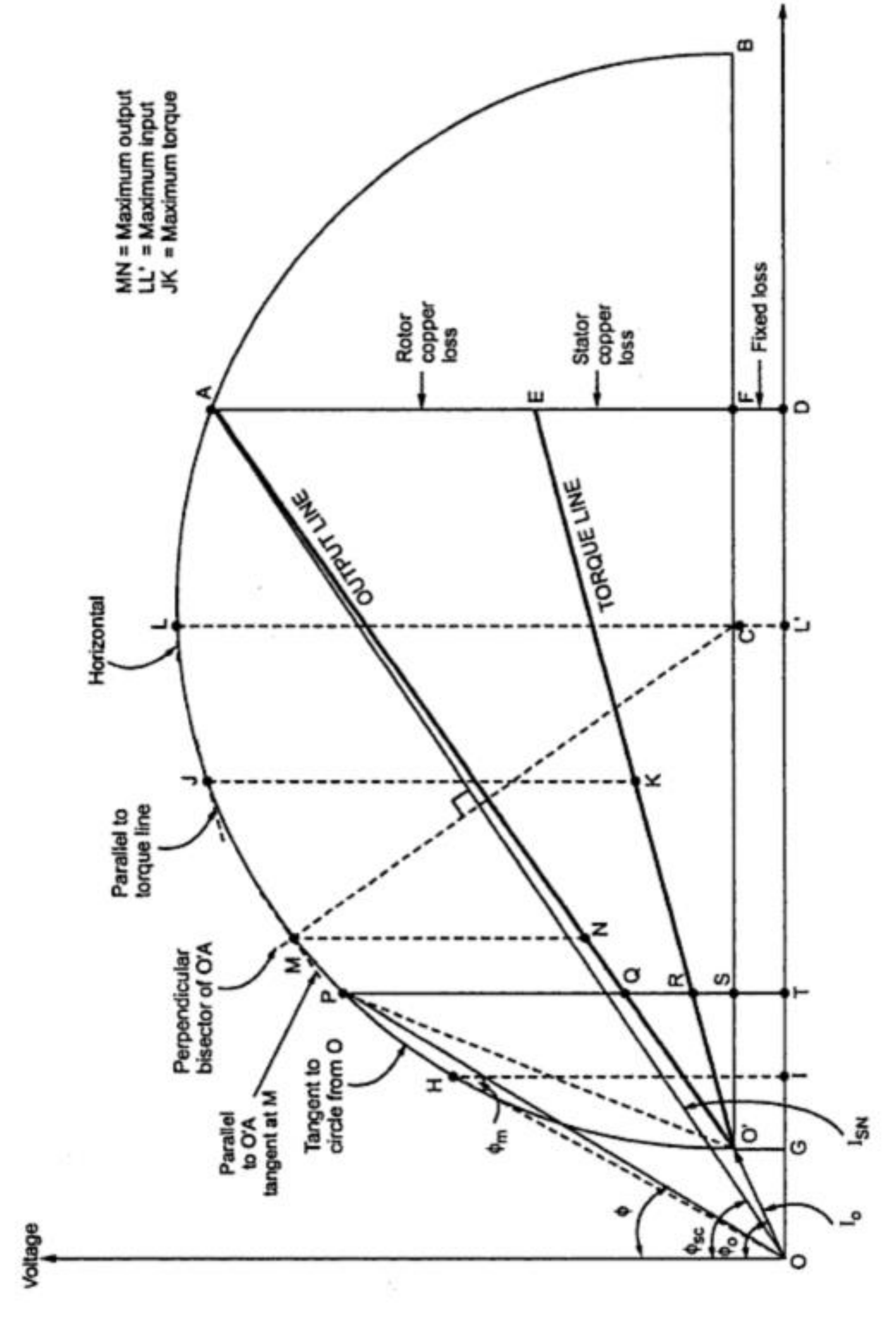

MN = Maximum output
LL' = Maximum input
JK = Maximum torque
Voltage
Horizontal
Parallel to torque line
Perpendicular bisector of O'A
Parallel to O'A tangent at M
Tangent to circle from O
Rotor copper loss
Stator copper loss
Fixed loss
OUTPUT LINE
TORQUE LINE
A
B
C
D
E
F
G
H
J
K
L
L'
M
N
O
O'
P
Q
R
S
T
φ
φm
φsc
φo
Io
ISN

Fig. 1

Power scale : As AD represents W_{SN} i.e. power input on short circuit at normal voltage, the power scale can be obtained as,

Power scale = $W_{SN}/l(AD)$ W/cm

where l(AD) = Distance AD in cm

Location of Point E : In a slip ring induction motor, the stator resistance per phase R_1 and rotor resistance per phase R_2 can be easily measured. Similarly by introducing ammeters in stator and rotor circuit, the currents I_1 and I_2 also can be measured.

∴ $K = I_1/I_2$ = Transformation ratio

Now   AF/EF = Rotor copper loss / Stator copper loss = $(I_2^2R_2)/(I_1^2R_1) = (R_2/R_2)(I_2^2/I_1^2) = (R_2/R_2).(1/K^2)$

But $R_2' = R_2/K^2$ = Rotor resistance referred to stator

∴ $AE/EF = R_2'/R_1$

Thus point E can be obtained by dividing line AF in the ratio R_2' to R_1.

In **a squirrel cage motor**, the stator resistance can be measured by conducting resistance tset.

∴   Stator copper loss = $3I_{SN}^2 R_1$ where I_{SN} is phase value.

Neglecting core loss, W_{SN} = Stator Cu loss + Rotor Cu loss

∴   Rotor copper loss = $W_{SN} - 3I_{SN}^2 R_1$

∴ $AE/EF = (W_{SN} - 3I_{SN}^2 R_1)/(3I_{SN}^2 R_1)$

Dividing line AF in this ratio, the point E can be obtained and hence O'E represents torque line.

Predicting Performance Form Circle Diagram

Let motor is running by taking a current OP as shown in the Fig. 1. The various performance parameters can be obtained from the circle diagram at that load condition.

Draw perpendicular from point P to meet output line at Q, torque line at R, the base line at S and horizontal axis at T.

We know the power scale as obtained earlier.

Using the power scale and various distances, the values of the performance parameters can be obtained as,

Total motor input = PT x Power scale

Fixed loss = ST x power scale

Stator copper loss = SR x power scale

Rotor copper loss = QR x power scale

Total loss = QT x power scale

Rotor output = PQ x power scale

Rotor input = PQ + QR = PR x power scale

Slip s = Rotor Cu loss = QR/PR

Power factor cos = PT/OP

Motor efficiency = Output / Input = PQ/PT

Rotor efficiency = Rotor output / Rotor input = PQ/PR

Rotor output / Rotor input = 1 - s = N/N_s = PQ/PR

The torque is the rotor input in synchronous watts.

Maximum Quantities

The maximum values of various parameters can also be obtained by using circle diagram.

1. Maximum Output : Draw a line parallel to O'A and is also tangent to the circle at point M. The point M can also be obtained by extending the perpendicular drawn from C on O'A to meet the circle at M. Then the maximum output is given by l(MN) at the power scale. This is shown in the Fig. 1.

2. Maximum Input : It occurs at the highest point on the circle i.e. at point L. At this point, tangent to the circle is horizontal. The maximum input given l(LL') at the power scale.

3. Maximum Torque : Draw a line parallel to the torque line and is also tangent to the circle at point J. The point J can also be obtained by drawing perpendicular from C on torque line and extending it to meet circle at point J. The l(JK) represents maximum torque in synchronous watts at the power scale. This torque is also called stalling torque or pull out torque.

4. Maximum Power Factor : Draw a line tangent to the circle from the origin O, meeting circle at point H. Draw a perpendicular from H on horizontal axis till it meets it at point I. Then angle OHI gives angle corresponding to maximum power factor angle.

∴ Maximum p.f. = cos ∟{OHI}

= HI/OH

5. Starting Torque : The torque is proportional to the rotor input. At s = 1, rotor input is equal to rotor copper loss i.e. l(AE).

∴ T_{start} = l(AE) x Power scale ....in synchronous watts

Full load Condition

The full load motor output is given on the name plates in watts or h.p. Calculates the distance corresponding to the full load output using the power scale.

Then extend AD upwards from A onwards, equal to the distance corresponding to full load output, say A'. Draw parallel to the output line O'A from A' to meet the circle at point P'. This is the point corresponding to the full load condition, as shown in the Fig. 2.

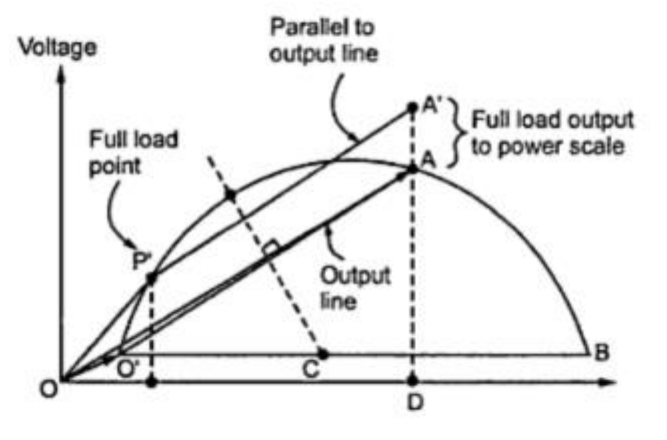


Fig. 2 Locating full load point

Once point P' is known, the other performance parameters can be obtained easily as discussed above.

Necessity of Starter

In a three phase induction motor, the magnitude of an induced e.m.f. in the rotor circuit depends on the slip of the induction motor. This induced e.m.f. effectively decides the magnitude of the rotor current. The rotor current in the running condition is given by,

$$\boxed{I_{2r} = \frac{sE_2}{\sqrt{R_2^2 + (s\,X_2)^2}}}$$

But at start, the speed of the motor is zero and slip is at its maximum i.e. unity. So magnitude of rotor induced e.m.f. is very large at start. As rotor conductors are short circuited, the large induced e.m.f. circulates very high current through rotor at start.

The condition is exactly similar to a transformer with short circuited secondary. Such a transformer when excited by a rated voltage, circulates very high current through short circuited secondary. As secondary current is large, the primary also draws very high current from the supply.

Similarly in a three phase induction motor, when rotor current is high, consequently the stator draws a very high current from the supply.

Similarly in a three phase induction motor, when rotor current is high, consequently the stator draws a very high current from the supply. This current can be of the order of 5 to 8 times the full load current, at start.

Due to such heavy inrush current at start there is possibility of damage of the motor winding. Similarly such sudden inrush of current causes large line voltage drop. Thus other appliances connected to the same line may be subjected to voltage spikes which may affect their working. To avoid such effects, it is necessary to limit the current drawn by the motor at start. The starter is a device which is basically used to limit high starting current by supplying reduced voltage to the motor at the limit of starting. Such a reduced voltage is applied only for short period and once rotor gets accelerated, full normal rated voltage is applied.

Not only the starter limits the starting current but also provides the protection to the induction motor against overloading loading and low voltage situations. The protection against single phasing is also provided by the starter. The induction motor having rating below 5 h.p. can withstand starting currents hence such motors can be started directly on line. But such motors also need overload, single phasing and low voltage protection which is provided by a starter.

Thus all the three phase induction motors need some or the other type of starter.

Types of Starters

From the expression of rotor current it can be seen that the current at start can be controlled by reducing E_2 which is possible by supplying reduced voltage at start or by increasing the rotor resistance R_2 at start. The second method is possible only on case of slip ring induction motors. The various types of starters based on the above two methods of reducing the starting current are,

1. Stator resistance starter

2. Autotransfomer starter

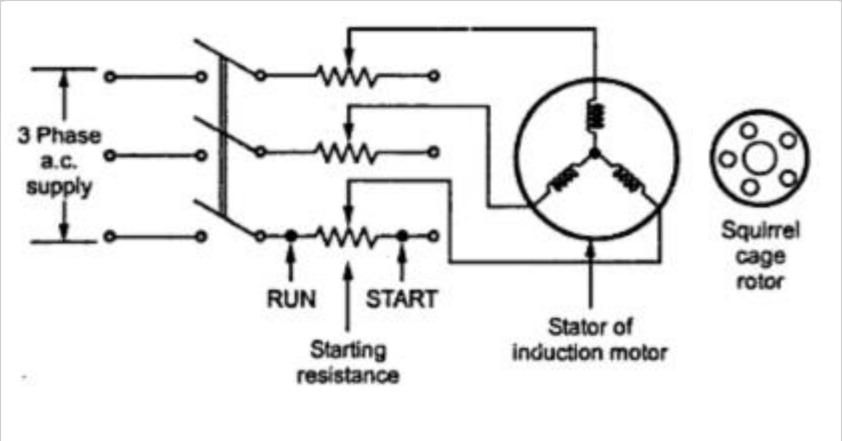
3. Star-delta starter

4. Rotor resistance starter

5. Direct on line starter

Stator Resistance Starter

In order to apply the reduced voltage to the stator of the induction motor, three resistances are are added in series with each phase of the stator winding. Initially the resistances are kept maximum in the circuit. Due to its large voltage gets dropped across the resistances. Hence a reduced voltage gets applied to the stator which reduces the high starting current. The schematic diagram showing stator resistances is shown in the Fig..1.



When the motor starts running, the resistances are gradually cut-off from the stator circuit. When the resistances are entirely removed from the stator circuit i.e. rheostats in RUN position then rated voltage gets applied to the stator. Motor runs with normal speed.

The starter is simple in construction and cheap. It can be used for both star and delta connected stator. But there are large power losses due to resistances. Also the starting torque of the motor reduces due to reduced voltage applied to the stator.

Relation between T_{st} and $T_{F.L.}$

We know, $P_2 = T \times \omega_s$

where T is torque produced and P_2 is the rotor input at N_s.

$\therefore \quad T \propto P_2$

But $\quad P_2 = P_c/s \quad$ where P_c = Total copper loss

$= (3I_{2r}^2R_2)/s$

$\therefore \quad T \propto I_{2r}^2/s$

But rotor current I_{2r} and stator current are related to each other through transformer action.

$\therefore \quad T \propto I_1^2/s \quad$ where I_1 = Stator current

At start, $\quad s = 1, \quad T = T_{st} \quad$ and $I_1 = I_{st}$

$\therefore \quad T_{st} \propto I_{st}^2 \qquad$ ....(1)

When stator resistance starter is used, the factory by which stator voltage reduces is say $x < 1$. The starting current is proportional to to this factor x. So if I_{sc} is the normal current drawn under full rated voltage condition at start then,

$I_{st} = x\, I_{sc} \qquad$ ....(2)

$\therefore \quad T_{st} \propto (xI_{sc})^2 \qquad$ ....(3)

But $\quad T_{F.L.} \propto (I_{F.L.})^2/s_f \quad$ where s_f = Full load slip $\qquad$ ....(4)

Taking ratio of (3) and (4),

$$\boxed{\frac{T_{st}}{T_{F.L.}} = x^2 \left(\frac{I_{sc}}{I_{F.L.}}\right)^2 s_f}$$

Note : As x < 1, it can be seen that the starting torque reduces by the fraction x^2due to the stator resistance starter.

Autotransformer Starter

A three phase star connected autotransformer can be used to reduce the voltage applied to the stator. Such a starter is called an autotransformer starter. The schematic diagram of autotransformer starter. The schematic diagram of autotransformer starter is shown in the Fig..1.

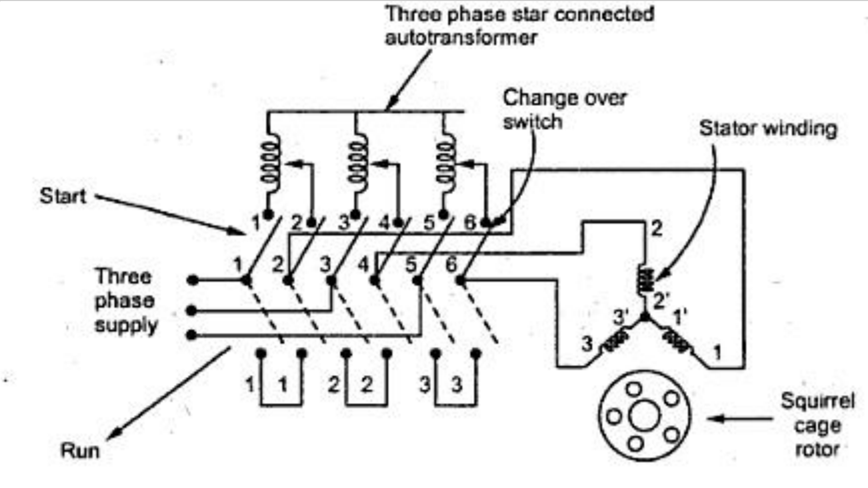


Fig. 1 Autotransformer starter

It consists of a suitable change over switch.

When the switch is in the start position, the stator winding is supplied with reduced voltage. This can be controlled by tappings provided with autotransformer.

The reduction in applied voltage by the fractional percentage tappings x, used for an autotransformer is shown in the Fig. 2.

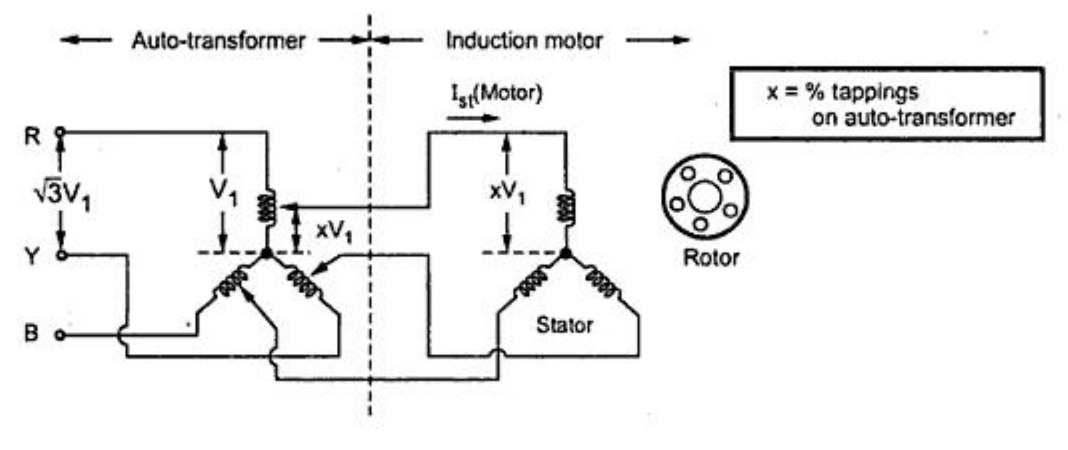


Fig.2 Use of auotransformer to reduce voltage at start

When motor gathers 80% of the normal speed, the change over switch is thrown into run position.

Due to this, rated voltage gets applied to stator winding. The motor starts rotating with normal speed. Changing of switch is done automatically by using relays. The power loss is much less in this type of starting. It can be used for both star and delta connected motors. But it is expensive than stator resistance starter.

Relation between T_{st} and $T_{F.L.}$

Let x be the fractional percentage tappings used for an autotransformer to apply reduced voltage to the stator.

So if, I_{sc} = Starting motor current at rated voltage

and I_{st} = Starting motor current with starter

then $I_{st} = x\, I_{sc}$ .....Motor side ....(1)

But there is exists a fixed ratio between starting current drawn from supply I_{st}(supply) and starting moor current I_{st} (motor) due to autotransformer, as shown in the Fig.3.

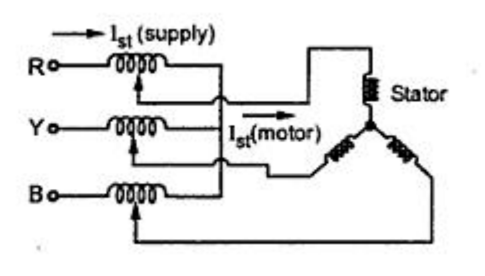


Fig. 3

Autotransformer ratio x = I_{st} (supply)/ I_{st} (motor)

I_{st} (supply) = x I_{st} (motor) ....(2)

Substituting I_{st} (motor) from equation (1),

∴ I_{st} (supply) = x . x I_{sc} = $x^2 I_{sc}$ ....(3)

Now $T_{st} \propto I_{st}^2 \text{(motor)} \propto x^2 I_{sc}^2$

and $T_{F.L.} \propto (I_{F.L.})^2/s_f$

∴ $$\boxed{\frac{T_{st}}{T_{F.L.}} = x2\left[\frac{I_{sc}}{I_{F.L.}}\right]^2 \times s_f}$$

Note : Thus starting torque reduces by x^2 where x is the transformer ratio.

Example : A squirrel cage induction motor has a full load slip of 5%. The motor starting current at rated voltage is 6 times its full load current. Find the tapping on the autotransformer starter which would give full load torque at start. Whet would then be the supply starting current ?

Solution : Starting current at rated voltage = I_{sc}

∴ $I_{sc} = 6\, I_{F.L.}$ and s_f = 5% = 0.05

Let x = Tapping on autotransformer

$T_{F.L.} = T_{st}$

∴ $$\boxed{\frac{T_{st}}{T_{F.L.}} = x2\left[\frac{I_{sc}}{I_{F.L.}}\right]^2 \times s_f}$$

$1 = x^2 (6/1)^2 \times 0.05$

$x = 0.7453$

Thus 74.53% tapping is required

Now I_{st} (supply) = x I_{st} (motor) = x (xI_{sc}) = $x^2 I_{sc}$

$= x^2 \times 6 = 3.33\ I_{F.L.}$

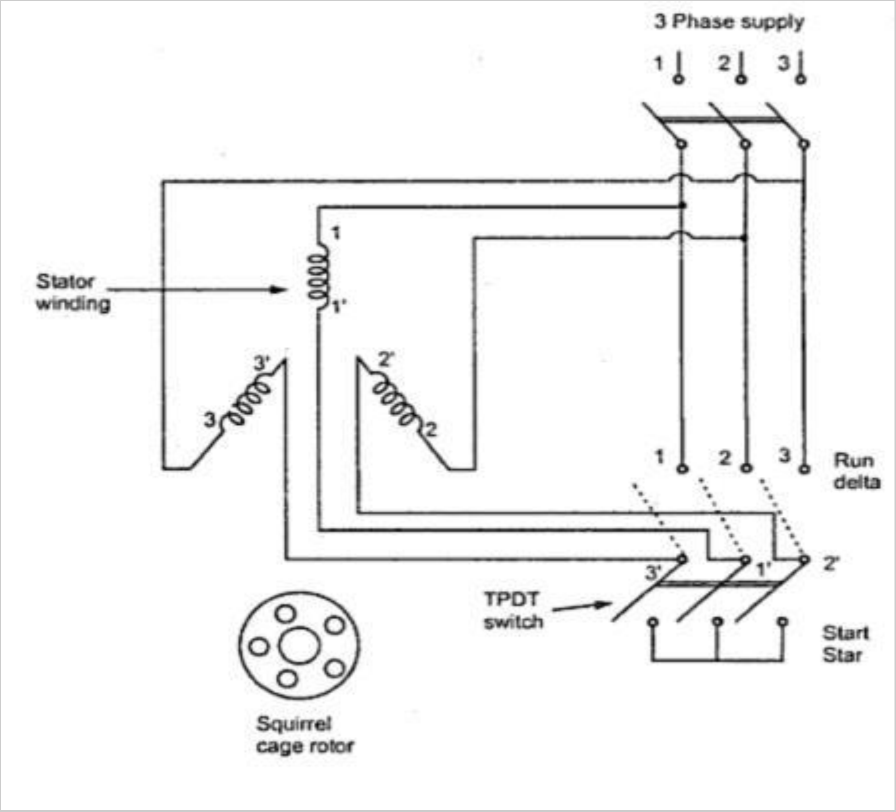
Thus supply starting current is 3.33 times the full load current.

Star - Delta Starter

This is the cheapest starter of all and hence used very commonly for the induction motors. It uses tripple pole double throw (TPDT) switch. The switch connects the stator winding in star at start. Hence per phase voltage gets reduced by the factor 1/√3. Due to this reduced voltage, the starting current is limited.

When the switch is thrown on other side, the winding gets connected in delta, across the supply. So it gets normal rated voltage. The windings are connected in delta when motor gathers sufficient speed.

The arrangement of star-delta starter is shown in the Fig.



The operation of the switch can be automatic by using relays which ensures that motor will not start with the switch in Run position. The cheapest of all and maintenance free operation are the two important advantages of this starter. While its limitations are, it is suitable for normal delta connected motors and the factor by which voltage changes is 1/√3 which can not be changed.

Ratio of T_{st} to $T_{F.L.}$

We have seen in case of autotransformer that if x is the factor by which the voltage is reduced then,

$$\therefore \qquad \boxed{\frac{T_{st}}{T_{F.L.}} = x^2 \left[\frac{I_{sc}}{I_{F.L.}}\right]^2 \times s_f}$$

Now the factor x in this type of starter is 1/√3.

$$\therefore \qquad \boxed{\frac{T_{st}}{T_{F.L.}} = \frac{1}{3}\left(\frac{I_{sc}}{I_{F.L.}}\right)^2 s_f}$$

where I_{sc} = Starting phase current when delta connection with rated voltage

$I_{F.L.}$ = Full load phase current when delta connection

Example : A three phase induction motor has a ratio of maximum torque to full load torque as 2.5 : 1. Determine the ratio of starting torque to full load torque if star-delta starter is used. The rotor resistance and standstill reactance per phase are 0.4 and 4 respectively.

Solution : The given ratio is, $T_m / T_{F.L.} = 2.5$

The rotor values are, $R_2 = 0.4\Omega \quad X_2 = 4\Omega$

Now $T_m = (kE_2^2)/(2X_2)$

$\therefore \quad T_{F.L.} = T_m/2.5 = (kE_2^2)/(5X_2) = (kE_2^2)/20$ ....(1)

Now $T_{st} = (k E_2^2 R_2)/(R_2^2 + X_2^2)$

With star-delta starter $E_2 = E_2/\sqrt{3}$

$$\therefore \qquad T_{st} = \frac{k\left(\frac{E_2}{\sqrt{3}}\right)^2 R_2}{R_2^2 + X_2^2} \qquad \ldots (2)$$

Taking ratio of (2) and (1),

$$\frac{T_{st}}{T_{F.L.}} = \frac{k\left(\frac{E_2}{\sqrt{3}}\right)^2 R_2}{R_2^2 + X_2^2} \times \frac{20}{k\,E_2^2} = \frac{20 \times 0.4}{3[(0.4)^2 + (4)^2} = 0.165$$

Speed Control of Three Phase Induction Motor

A three phase induction motor is practically a constant speed motor like a d.c. shunt motor. But the speed of d.c. shunt motor can be varied smoothly just by using simple rheostats. This maintains the speed regulation and efficiency of d.c. shunt motor. But in case of three phase induction motors it is very difficult to achieve smooth speed control. And if the speed control is achieved by some means, the performance of the induction motor in terms of its power factor, efficiency etc. gets adversely affected.

For the induction motor we know that,

$$T \propto \frac{sE_2^2\,R_2}{R_2^2 + (s\,X_2)^2}$$

From this expression it can be seen that the speed of induction motor can be changed either by changing its synchronous speed or by changing the slip s.

Similarly torque produced in case of three phase induction motor is given by,

$$N = N_s\,(1 - s)$$

So as the parameters like R_2, E_2 are changed then to keep the torque constant for constant load condition, motor reacts by change in its slip. Effectively its speed changes.

Thus speed of the induction motor can be controlled by basically two methods :

1. From stator side and

2. From rotor side

From stator side, it includes following methods :

a. Supply frequency control to control N_s, called V / f control.

b. Supply voltage control.

c. Controlling number of stator poles to control N_s.

d. Adding rheostats in stator circuit.

From rotor side, it includes following methods :

a. Adding external resistance in the rotor circuit.

b. Cascade control.

c. Injecting slip frequency voltage into the rotor circuit.